The Latent Path Model for Dynamic Social Networks with an Application to Party Switching in Poland*

Jason W. Morgan†

September 15, 2015

Abstract

High rates of party switching by politicians is often expected to inhibit party system institutionalization by reducing democratic representation, accountability, and the heuristic value of party labels. However, in new democracies, where social connections and party labels are weak, switching may allow politicians the flexibility to sort themselves into more cohesive groups, ultimately contributing to an increased likelihood of long-term party system stability. To investigate the phenomenon of party switching more closely, this paper develops a new latent variable model suitable for analyzing dynamic network data. The proposed latent path model is a natural extension of the latent space model for static networks developed by Hoff, Raftery, and Handcock (2002) and is in the spirit of the dynamic network model of Ward, Ahlquist, and Rozenas (2013). An application of the model to party switching in Poland, which has seen more than 1,100 instance of party switching since the first democratic election in 1991, shows that switching during the first five parliamentary terms resulted in greater ideological coherence of parties and that a core group of 199 long-serving MPs has been at the leading edge of this convergence. This counterintuitive result suggests party switching may sometimes play a more constructive role in party system institutionalization than typically realized, while it also suggests that the Polish party system may be developing the foundations of a strong and stable party system.

*I would like to thank Jan Box-Steffensmeier, Paul DeBell, Luke Keele, Austin Knuppe, William Minozzi, Irfan Nooruddin, Santiago Olivella, Andrew Rosenberg, and Peter Tunkis for their numerous helpful comments. This project was supported by the Ohio Supercomputer Center and the Institute for Population Research at The Ohio State University. Earlier versions were presented at the Society for Political Methodology's Annual Summer Methods Meeting (2013–2014) and the Annual Meeting of the Political Networks Section of the American Political Science Association (2013–2014).

†Post-Doctoral Researcher, Department of Political Science, The Ohio State University. Email: morgan.746@osu.edu.

1 Introduction

Scholars have long viewed a stable party system to be an important component of a healthy democracy (Huntington 1968; Schattschneider 1942). A key to developing such a party system is the development of stable partisan attachments between politicians and voters (Converse 1969; Mainwaring 1999; Mainwaring and Scully 1995). Consequently, high rates of party switching by politicians is often seen as a clear indicator of a lack of overall party system institutionalization. Party switching can reduce politicians' accountability to voters, suggest a lack of party organization and discipline, and lessen the heuristic value of party labels for voters (Desposato 2006; Heller and Mershon 2005, 2009; Mainwaring 1998).

The empirical evidence suggests that party switching is damaging to the process of party system institutionalization in young democracies. When politics are uncertain and voters have yet to learn the contours of the new democratic regime, constant switching frustrates attempts by voters to hold their elected officials accountable for poor performance (Zielinski, Słomczynski, and Shabad 2005). Furthermore, high levels of switching can contribute to disorganization in parliament, overall party system fragmentation (Kreuzer and Pettai 2003), and has been shown to encourage the self-serving goals of politicians (Desposato 2006). As Desposato (2006, p. 77) argues, "[s]witching effectively destroys the meaning of party labels, raises voters' information costs, and eliminates party accountability."

There are, however, theoretical reasons to believe that party switching per se may not be inherently detrimental to party systems. As Heller and Mershon point out, the notion that switching is damaging to democratic representation assumes that voters select candidates based solely on their party affiliation. In other words, party labels are meaningful and, as such, provide a great deal of valuable information to voters about candidates' and parties' policy positions (Heller and Mershon 2009). Yet, switching may play a constructive role if it provides politicians the flexibility to take positions on policy that more closely match the views of their constituents.[1] Likewise, when labels carry little meaning, as is the case in new democracies, switching may play a constructive role by allowing politicians the freedom

[1] As an empirical example, consider the partisan realignment in the U.S. after the passage of the Civil Rights Act in 1964, which made the party labels more closely reflect the values of the voters in the South (Levendusky 2009).

to sort themselves into more cohesive groups that reflect the ideological contours of society, thus ultimately contributing to an increased likelihood of long-term party system stability.

In this paper, I take a dynamic network approach to assessing whether patterns of party switching in Poland indicate a growing coherence of parties in the country or whether this switching indicates continued party system weakness. Poland makes an interesting and difficult case for the proposition that party switching can play a positive role in party system development. Ever since the first democratic parliamentary election in 1991, the Sejm has been plagued by a chronic tendency for elected members of parliament to switch parties. During the first five parliamentary terms, there were nearly 1,100 instances of intra-term party switching in the Sejm, the lower house, with almost 30% of members of parliament (MPs) switching parties at least once. As a consequence of this switching, more than 70 different parties served in parliament during this period. The fragile nature of elite partisan attachments has contributed to persistent government instability: during the first seven parliamentary terms, Poland had 17 different governments, 11 prime ministers, and only one government survived to complete a full four-year term (Conrad and Golder 2010).[2]

By other common measures of party system institutionalization, however, the Polish party system has begun to show some encouraging signs of stabilization. For instance, despite the upheaval caused by MP party switching, only one new party has been elected to the Sejm during each of the last three electoral cycles. Furthermore, in 2011, Civic Platform (PO) won its second consecutive election, a first for Poland, nearly duplicating its 2007 performance. At the same time, Law and Justice (PiS) retained its position as the main opposition. This greater electoral certainty is reflected in declining electoral volatility and low levels of fragmentation, and seems to suggest that an enduring and meaningful division has emerged in Polish politics.[3]

From the perspective of extant theory, as noted above, the Polish party system raises interesting questions about the state of institutionalization in the country and about the process of party system institutionalization in new democracies more generally. Long established

[2]These figures assume the present governing coalition between Civic Platform (PO) and the Polish Peasant Party (PSL) survives the current term that ends in 2015.

[3]Szczerbiak (2013) notes that this division appears to be real, with PO and PiS having "become the main points of reference for each other" (Szczerbiak 2013, pp. 493–494).

theories of party system development hold that the advent of stable partisan commitments on the part of politicians and voters is critical to the process of institutionalization (Huntington 1968). At the same time, these theories emphasize the importance ideology plays in structuring party competition, with stable party systems exhibiting a close identification between particular parties and ideological positions (Mainwaring and Scully 1995). The level of party switching seen in Poland clearly suggests a lack of party system institutionalization; however, more recent stability in the partisan makeup of electoral politics, and the relatively stable ideological positions of those parties (Markowski 2008; Szczerbiak 2013), suggests growing coherence in the party system.[4] What explains the discrepancy between politics at the elite level and that at the aggregate electoral level? Why has not the lack of partisan commitments by members of parliament and resultant intra-term party system instability translated into even greater party fragmentation? Finally, what can past patterns of party switching tell us about the possibility of party system institutionalization in Poland?

To date, party switching has been treated as an individual-level phenomenon, whereby switching as an outcome is determined by legislators' perceptions of the costs and benefits associated with doing so (Desposato 2009; Heller and Mershon 2005; Laver and Benoit 2003; Zielinski, Słomczynski, and Shabad 2005). These benefits of switching include increasing the likelihood of being reelected, obtaining rent from holding office, or achieving some policy outcome. While these approaches are focused on individual-level decision making, there is an implicit relational aspect to the theories in that the benefits to be gained from switching is conditional on the rest of the party system remaining constant. Empirically, these approaches have assumed such independence. However, there are reasons to believe that such independence does not exist. For instance, a party switch by one MP may increase the likelihood that allies in her old party switch in the future; party dissolution may make it necessary for many MPs to "switch" parties simultaneously; and, in a new party system, a learning process may occur which decreases the probability of switching over time, perhaps due to solidifying partisan lines or increasing party discipline. In other words, party switching should be seen as a relational, dynamic process; thus, empirical analyses seeking

[4] Looking at an earlier period, Shabad and Słomczynski (2004) also note the presence of switching alongside indicators of party system institutionalization.

to understand this process require the use of models appropriate for such data.

In this paper, I develop a new latent variable model suitable for tracing the movement of members of parliament through a latent social space over time. The proposed model, which I call the latent path model, builds on the latent space models of Hoff, Raftery, and Handcock (2002) and is similar to the dynamic latent space model of Sewell and Chen (2015).[5] The model differs, however, from prior research in that it allows the explicit modeling of non-linear trends in the movement of actors in the latent social space; can accommodate directed, undirected, and weighted networks; and fits more neatly into generalized linear models familiar to political scientists. I provide a Bayesian implementation of the model in Stan (Stan Development Team 2013). I then apply the proposed latent path model to a core subset of 199 Polish MPs. I find that switching during this period, rather than being a symptom of continued party system fragmentation, has resulted instead in greater ideological coherence of parties. In other words, switching seems to have played a constructive role in the Polish party system by allowing politicians the flexibility to sort themselves into more ideologically homogeneous parties. Overall, these counterintuitive results suggest that party switching may not necessarily be a detriment to party system institutionalization and democratic consolidation more generally. Furthermore, from a broader theoretical perspective, the patterns of change seen in Poland provide insight into the process through which ideologically homogeneous and stable parties develop from the dynamic interaction of politicians in new democracies.

In the next section, I briefly describe the overall trends in party politics in Poland since the democratic transition in 1991, while also providing a more detailed discussion of party switching in the Polish Sejm from a network perspective. Section 3 discusses the problems posed by relational data, such as the party switching network, for standard statistical models, and introduces the latent space model previously developed for static networks (Hoff, Raftery, and Handcock 2002). In Section 4, I then propose a latent space model for dynamic networks. This model is applied to the Polish party switching data in Section 5. Section 6 concludes.

[5] Also see Sarkar and Moore (2005) for an earlier development along these lines.

2 Poland's Party System

Compared to other third wave democracies, party systems in post-communist Eastern Europe have been slow to institutionalize. Overall, these party systems can be characterized by their comparatively high levels of fragmentation, electoral volatility, and general uncertainty (Bakke and Sitter 2005; Bielasiak 2002; Epperly 2011; Lewis 2000). At first glance, this volatility is unsurprising given the unique obstacles these societies faced in their efforts to shed the economic and social legacies of communism (Mair 1997, ch. 8; Offe 1993). However, by many measures, it remains unclear whether party systems in these countries are moving in the right direction and whether scholars can yet talk about general trends in party system institutionalization in the region. On the one hand, there have been some positive signs that "democratic maturation" is occurring (Tavits 2005; Tavits and Annus 2006), and expected patterns of economic voting are emerging (Duch 2001; Tucker 2006). On the other hand, overall indicators of institutionalization suggest that the consistent patterns that we would usually expect from institutionalized systems have not yet emerged (Casal Bértoa and Mair 2012).

The party system in Poland has been particularly resistant to stabilization. In many ways, this is surprising. In contrast to other countries that saw the thorough flattening of society by the communist regimes, Poland managed to maintain some semblance of civil society,[6] as evinced by the importance of Solidarity in the democratic transition, while also preserving a largely autonomous Catholic Church and resisting large-scale collectivization of the agricultural sector. Furthermore, two communist successor parties—the Democratic Left Alliance (SLD) and the Polish Peasants Party (PSL)—survived the democratic transition relatively intact.[7] These parties managed to maintain their organizational structure and were headed by long-standing members (Grzymała-Busse 2002), which lent a degree of pre-existing structure to the party system.[8]

[6]Of course, the communist period did not result in perfectly homogeneous societies. As Słomczynski and Shabad point out, making this assumption "obscures the nature, degree, and consequences of social differentiation in these societies, both before and after the onset of systemic change" (Słomczynski and Shabad 1996, p. 188).

[7]SLD was the successor to the communist-era ruling Polish United Workers' Party (PZPR), while PSL was the successor to United People's Party (ZSL), a communist era agrarian satellite party.

[8]Strictly speaking, SLD did not consolidate into a single party until 1999, when Social Democracy of the

By anchoring the political spectrum ideologically, the presence of Solidarity and the Catholic Church on the right and SLD and PSL on the left should have aided Poland in developing a robust and stable party system. But this has not been the case. Instead, the party system in Poland has been characterized as being "completely under-institutionalized" (Casal Bértoa 2012, p. 5), which is reflected in Poland having some of the lowest levels of partisan attachment in post-communist Europe (van Biezen, Mair, and Poguntke 2012; Whiteley 2011), persistently high levels of electoral volatility (Epperly 2011), and some of the highest turnover in governing coalitions in the region (Casal Bértoa and Mair 2012; Conrad and Golder 2010; Grotz and Weber 2012). Poland also has the lowest level of turnout in post-communist Europe, averaging 47.7% in national parliamentary elections. Only two elections (in 1993 and 2005) recorded turnout above 50%, and the first fully democratic election in 1991 recorded a turnout of just 43.2%, an astonishingly low figure given Poland's leading role in the regional transition. By comparison, in the Czech Republic and Hungary, turnout averaged over 70% and 60% during the same period (Birch 2003, pp. 60–61; Kostadinova 2003).[9]

None of this is to say that Poland has not made noticeable progress towards developing a more stable party system. Some clear and positive indicators are available. Table 1 presents some general electoral trends in Poland over the last two decades. From the data presented in this table, it is tempting to say that the Polish party system has settled into a relatively stable pattern of party competition. For example, as measured by the effective number of parties serving in parliament it appears that the party system has resisted extreme levels of fractionalization.[10] Furthermore, only one new party has been elected to the parliament in each of the last three elections, which indicates that existing parties are beginning to attract stable levels of support and themselves becoming more institutionalized. Finally, in the last two elections, the same ruling party, center-right Civic Platform (PO), has won and formed

Republic of Poland (SdRP) and the Polish Social Democratic Union (PUS) merged. They mostly competed as a single entity in elections prior to this, however.

[9]In the partially-free election of 1989 turnout was somewhat over 60%, though this was still lower than the first elections throughout post-communist Europe (Kostadinova 2003).

[10]In the first democratic election, there was no electoral threshold, which resulted in a large number of parties (29) winning seats in the Sejm. A 5% threshold was instituted for the second election in 1993, which contributed significantly to the decline in the number of parties in parliament.

Table 1: Overview of Polish Parliamentary Elections Results, 1991–2011.

	1991	1993	1997	2001	2005	2007	2011
Total elected	29	8	6	7	7	5	6
New elected	–	6	5	4	1	1	1
Total competing	52	26	25	14	22	10	11
Effective parties	11.3	3.8	2.9	3.6	4.2	2.8	2.9
Electoral volatility	–	41.6	64.8	54.4	35.3	34.1	12.3
Turnout (%)	43.2	54.0	47.9	46.3	40.3	53.8	48.9
Vote share top two parties (%)	24.3	39.9	60.7	53.7	51.1	73.6	69.1

Notes: Electoral volatility comes from Powell and Tucker (2014) and corresponds to their Total Volatility measure. Volatility for 2011 was calculated by the author. Effective number of parties calculated as in Laasko and Taagepera (1979).

the same coalition with PSL.[11]

2.1 Party Switching as a Network in Poland

As presented in the previous section, there is some question as to whether or not the Polish party system has been making progress towards a more stable pattern of competition. Traditional measures of party system institutionalization are rather ambiguous: the effective number of parties in parliament has been relatively stable since 1993; electoral volatility has declined since the 1997 election, and in the 2011 election was half the level it was during the previous election of 2007; and for the first time in post-communist Poland, the same coalition of PO and PSL won in two consecutive elections. However, one of the clearest signs of a lack of institutionalization in Poland has been the extreme fluidity and lack of stable partisan attachments at the elite level (Shabad and Słomczynski 2004). Table 2 shows the overall trend of party switching during the first five parliamentary terms in the Sejm.[12] During the period covered in the table, there were almost 1100 instances of intraterm party switching. There have been more than 100 switches during each term, with the lowest number of switches (105) occurring during the 2005 term and the most (481) during the previous 2001

[11] Gwiazda (2009) considers the Polish party system to be "quasi-institutionalized".

[12] The party switching data discussed here come from McMenamin and Gwiazda (2011). In their analysis, they provide an event history analysis of party switching in each term, with the objective of identifying the individual motivations for switching. These data are discussed further below.

Table 2: Number of Active Parties, MPs, and the Number of Switches by Term in the Polish Sejm, 1991–2005 Parliamentary Terms.

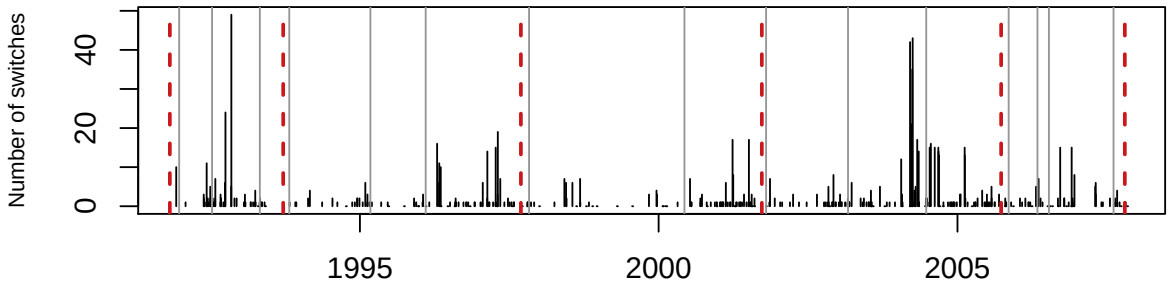
	1991	1993	1997	2001	2005
Parties elected	29	8	6	7	7
Parties existing	28	23	14	21	13
MPs	457	477	480	486	476
Switches	177	180	142	481	105
Number switched	128	70	93	176	39
Pct. switched	27.9	14.7	19.3	36.1	8.2
Max. MP switches	4	8	6	10	6

Source: McMenamin and Gwiazda (2011). These data include switches to unregistered status.

term. On average, more than 21% of MPs have changed their party affiliation at least once during each term, and of the 1603 MPs that have served in the Sejm over the period, 28.5% have switched parties at least once during their time in office. Another characteristic of this switching is that when MPs have changed their party affiliation, it was often to new parties instead of existing parties. Consequently, 74 different parties have served in the Sejm during this period.

Figure 1 provides another perspective on MP party switching in the Polish Sejm, reporting the number of changes in party affiliation by day over the first five parliamentary terms. The dashed vertical lines mark the dates of parliamentary elections, while the solid lines delineate the formation of a new government as reported by Conrad and Golder (2010, table 8, p. 143). Two things are remarkable about the patterns shown in this figure. First, while there have been some significant spikes in switching—which coincided with reconfigurations of major parties—switching is not constrained to such periods of acute volatility; instead, switching has been a continual feature of politics in the Sejm, with the only extended lull occurring in 1999. Second, while the summary of switching included in Table 2 seems to indicate that, with 105 total switches and 8.2% of MPs switching during the term, there was a decline in switching during the 2005 term, Figure 1 clearly shows that this decline in apparent switching is an artifact of the term being limited to two years. If the rate of switching is extrapolated out to a full four-year term, we would expect upwards of 200 switches during the term.

Figure 1: Number of Changes in Party Affiliation by Day in the Polish Sejm, 1991–2005 Parliamentary Terms.



Here I investigate this question with a descriptive analysis of party switching in the Polish Sejm. Unlike other studies that have looked at party switching from the perspective of the incentives facing individual politicians (Desposato 2006; Heller and Mershon 2005; Laver and Benoit 2003; Mershon and Shvetsova 2008), I take a social networks approach, which emphasizes the relational nature of party membership in parliaments.

The data for the following analyses come from McMenamin and Gwiazda (2011), and were collected directly from the records of the Polish parliament.[13] These data are unusually detailed, covering all intra-term party switching by MPs, including the exact day they switched as well as their destination party,[14] during the first five parliamentary terms of the Polish Sejm (Oct. 1991–Oct. 2007). Importantly, and what makes this dataset so interesting, is that it includes switches to and from parties that never competed formally in elections. It was not uncommon during this period for groups of MPs to leave their party

[13]Email correspondence with Anna Gwiazda (2013-03-11). In their original study, McMenamin and Gwiazda (2011) analyzed the data by term; consequently, some additional processing was needed before it could be analyzed as a single dataset. This included nomalizing the names of all politicians across time periods so that complete histories of switching could be constructed for each MP. Some additional cleanup of the data was also required. All data and detailed notes on the changes made to it are available upon request.

[14]MPs are allowed to be unregistered, and it was quite common for MPs to abandon their party and remain unaffiliated for some time, though it was not possible to get elected without a party affiliation. In this analysis, I ignore unregistered members. Thus, if two MPs from different parties leave their respective parties, they are not considered to share a party of unregistered MPs.

and form a new one, only to merge with another party at a later date. Such ephemeral parties would not be included in analyses that only looked at records of switching at the time of elections. However, these short-lived parties should be interesting to scholars interested in the dynamic evolution of social relations in the Sejm. These short-lived parties carry important information about personal allegiances and ideological subfactions present in parliament. Indeed, it is common for party subfactions to express their displeasure with their party by splitting from it. Looking just at the aggregate trends in party switching over the first five parliamentary terms, as was done in Section 2, it is easy to be pessimistic about the state of party system institutionalization in Poland. However, such summaries do little to reveal the structure of social relations between MPs in parliament. This is where a network approach begins to demonstrate its value.

Figure 2 presents the network of relationships between all Polish MPs and political parties as it is derived from the pattern of MP party switching in the Sejm.[15] During this period, 1603 MPs served and 74 parties operated in parliament. For interpretive clarity, four of the major parties are highlighted: SLD (red), PSL (green), PiS (blue), and PO (orange). All other parties are plotted as the larger dark-gray nodes and individual MPs are presented as small, light-gray nodes. As these are plotted as 2-mode, affiliation networks, ties represent MP membership in the parties during each of the terms. Thus, when an MP is tied to multiple parties, this indicates that the MP switched parties at least once during that term.[16]

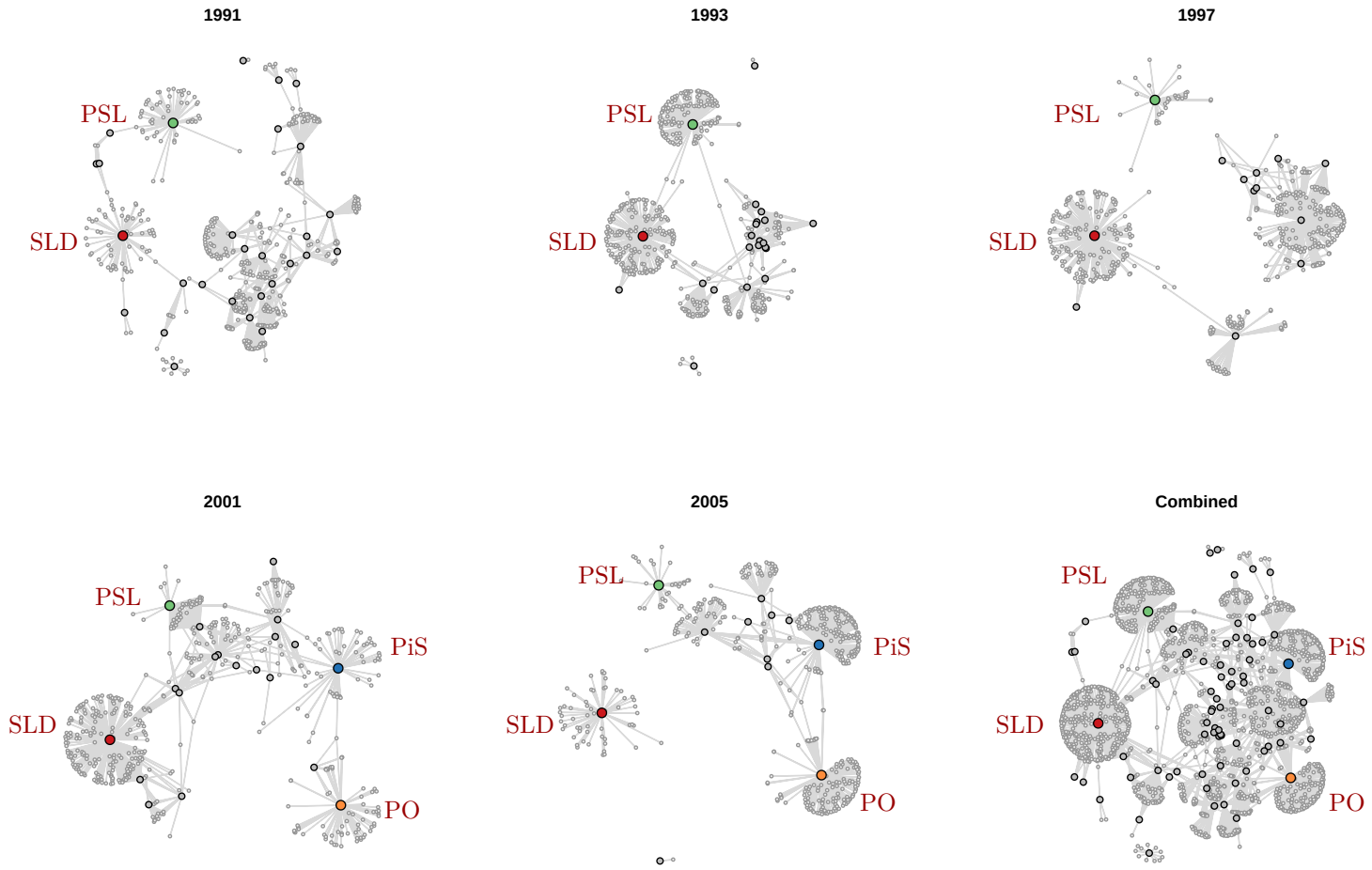
These network plots provide an interesting perspective on party politics in Poland. The

[15] The networks depicted in Figure 2 were laid out using the following procedure. First, locations for all nodes in the full network dataset were calculated using the algorithm of Fruchterman and Reingold (1991). This force-directed algorithm identifies positions to minimize edge overlap and distribute the nodes relatively evenly across the plot surface, while also maintaining the structure of the network. Second, the node locations were rotated so that SLD, the post-communist successor party, was to the left of PO and PiS. This conforms to what scholars know about these parties. Finally, to facilitate comparison across time periods, node locations in the full network graph where used to position the nodes in the specific time periods; in other words, MP and party node locations are static across time. It is important to emphasize that the node positions and the magnitude of distances between nodes depicted in the graphs are not meant to be used for inference.

[16] For the purpose of this analysis, all switches were treated equally; in other words, I did not distinguish between, for example, switches by individual MPs to other existing parties or induced by party splits. Other scholars have emphasized the importance of the different types of switches (Kreuzer and Pettai 2003; Shabad and Słomczynski 2004). Furthermore, multiple switches between parties are not accounted for. For example, if an MP switches from party A to party B and then back to party A, the data as analyzed simply record this as the MP having a membership tie to each of those parties. Future research, however, may be able to take advantage of the sequencing of changes in ties and the direction of the switching.

first thing that is readily apparent from the graphs is that SLD, the post-communist successor party, has managed to remain relatively cohesive during this period. This is clear from the party's relative isolation and lack of connections to other parties in the overall network. In other words, members of SLD have been less likely to switch parties, and the party has not suffered from the number of party splits that many of the parties on the right have experienced. Indeed, the lack of partisan commitments by politicians on the right is readily apparent in the graphs. Members of right-leaning parties in the Sejm have been much more likely to switch parties and the parties themselves have been much more susceptible to splits. This difference between the stability of the left and right is something that has been observed by other scholars (McMenamin and Gwiazda 2011).

Figure 2: Networks of Party Switching for each Parliamentary Term: 1991–2005. Large gray nodes indicate parliamentary parties, small gray nodes are individual MPs. Major parties are labeled and indicated by color: SLD (red), PSL (green), PiS (blue), and PO (orange).



Second, the network plots also reveal the growing association of PSL with the right side of the political spectrum. Over the first three parliamentary terms, PSL remained relatively isolated in terms of the party switching network, with the majority of switching that did occur being between SLD and PSL. This was likely a symptom of members of PSL, a successor party, maintaining social and ideological ties to SLD in the years following the collapse of the communist regime. Beginning in the 2001 term, however, the majority of switching PSL has experienced has been with the right. This change likely reflects two things. First, turnover in PSL's membership has meant the number of personal ties between members of PSL and SLD have declined. Second, PSL has been a party of opportunity, playing the role of pivotal party in parliament. Indeed, PSL has been a member of the governing coalition in 5 of 7 parliamentary terms—7 of 17 governing coalitions—since 1991 (Conrad and Golder 2010, p. 143, table 8).

Despite the usefulness of these network graphs in providing a general idea about how the party system in Poland has evolved, they are of limited use for determining whether any apparent patterns in party switching are indicative of growing coherence in the party system. For one thing, because of the high levels of party switching in Poland, the networks are simply too cluttered to allow for anything but the broadest patterns to be readily discernable, and even then the plots do not allow for any sort of formal inference to be performed. For another thing, while the overall orientation of the nodes has been specified in a way that makes sense from an ideological standpoint, the distance between pairs of nodes is not meaningful. In fact, for the purpose of presentation, the algorithm that positions the nodes intentionally limits the amount of node overlap.[17] Substantively, this would incorrectly suggest that two politicians could never hold the same position in the latent social space, which is an assumption that would be violated if any two MPs had the same pattern of party affiliation (something that is quite common in the Polish data). In the following sections, I develop a model capable of rigorously analyzing dynamic, relational data.

[17]Details of how the nodes were positioned are provided in fn 15.

3 The Latent Space Model

In this section, I present a discussion of network data and the problems such data pose for standard statistical models. I follow with a review of the latent space model for social networks, which provides the foundation for the dynamic network model I propose in Section 4.

3.1 Networks: Terminology and Representation

A network is a collection of actors and possible pairwise relations between those actors.[18] The simplest networks consist of a single type of actor, where ties are binary and non-directional. For example, in a network of international trade agreements, the actors would be states and the ties would represent the existence of a trade agreement between each state in the dyad. Directed ties are also possible. In directed networks, asymmetric relationships are possible. In a network of friendship ties, for instance, one member of a dyad may indicate a friendship with the other person, but this friendship may not be reciprocated. Networks need not be restricted to a single type of actor. More complex networks can include multiple types of actors with valued ties between them. In the Polish party switching network analyzed below, there are two types of actors—members of parliament and a party—where ties represent MP membership in the parties. In this case, ties are undirected. Networks with this type of structure are known as affiliation networks.

Mathematically, a network can be represented by a matrix, Y, known as an adjacency (or socio-) matrix. Each element of the matrix corresponds to a relationship between two actors in the network. In a simple network with a single type of N actors and binary relations, Y is an $N \times N$ matrix, with each element of the matrix, $y_{ij} \in \{0, 1\}$, indicating the existence of a tie between actors i and j.[19] In an undirected network, Y is symmetric; i.e., $y_{ij} = y_{ji} \; \forall \; i \neq j$. In a directed network, symmetry need not hold. In these networks, i represents the sender of a relationship, while j is the receiver. Weighted networks are also possible. In this case,

[18] Actors in the network are also known as nodes or vertices, while relations are known as ties or edges. The canonical introduction to network methods is Wasserman and Faust (1994), while Kadushin (2012) provides a more current review of a range of substantive applications.

[19] Self-ties, or loops, are not typically allowed and diagonal elements of the adjacency matrix are zero by definition; i.e., $y_{ii} = 0 \; \forall \, i$.

y_{ij} can take on any value.

Statistical analysis of network data focuses on explaining patterns of ties between actors, either at the dyadic level or from a broader structural perspective. Such data are often expected to have strong interdependencies. For instance, ties received by an actor are often reciprocated (reciprocity), actors with similar characteristics are more likely to have ties with each other (homophily), and friends of a friend are also more likely to be friends (transitivity). These interdependencies complicate the analyses of network data with standard statistical approaches, as they violate the common assumption in regression modeling that observed outcomes are independent conditional on the model and the included covariates. Consequently, by using logistic regression to model tie formation in binary networks, for example, while ignoring strong degrees of dependence in the process that generates these ties, scholars risk significant bias in estimates of coefficient and standard errors. Standard models are simply not appropriate for data with high levels of dependence between ties.

3.2 The Latent Space Model for Network Data

The methodological complications network data pose have encouraged the development of numerous statistical approaches to analyzing such data. Methods range in approach from actor-based, decision-theoretic models, which explain the observed network structure as the result of the cumulative decisions of actors in the network (Snijders, Bunt, and Steglich 2010), to more holistic modeling strategies like the exponential random graph model (ERGM), which aims to estimate the likelihood of observing a network in its entirety given particular structural characteristics (see Cranmer and Desmarais 2011, p. 222).

The latent space approach to modeling network data, first proposed by Hoff, Raftery, and Handcock (2002), takes something of a middle ground between the actor-based and holistic ERGM approaches. This model takes a network and posits that the presence (or strength) of a tie between each pair of actors in the network is a function of their positions in a latent social space. The fundamental assumption of this model is that actors located more closely together in the latent social space are more likely to have ties with each other. For example, in the application to party switching in the Sejm, the latent space could be interpreted as an ideological space, where MPs near each other in that space are more likely to share

parties.[20] Viewed this way, the latent space model is analogous to many ideal point models more commonly seen in political science (Poole and Rosenthal 1997; Clinton, Jackman, and Rivers 2004). In these models, a vote for a bill is seen to be more likely when it reflects a point close to a legislators' ideal policy position; thus, legislators that often vote the same way are seen to have similar positions on some ideological scale.[21]

Some mathematical notation should help further clarify things. As discussed above, a network can be defined as a set of pairwise ties between actors in the network. This set of ties defines a response vector, y_{ij}, where each element of the vector indicates whether or not there is a tie between actors i and j. In the latent space model, this response vector is modeled as a function of the pairwise distances between actors:

$$y_{ij} = \mathcal{F}\{\beta^T \mathbf{x}_{ij} - d(\mathbf{z}_i, \mathbf{z}_j)\}. \tag{1}$$

Here, $\mathbf{z}_i$ and $\mathbf{z}_j$ represent the k-dimensional vector of positions for actors i and j in the latent social space, while $d(\mathbf{z}_i, \mathbf{z}_j)$ is some distance function specified by the analyst that satisfies the triangle inequality. To ease interpretation, the Euclidean distance is often used; e.g.,

$$d(\mathbf{z}_i, \mathbf{z}_j) = \sqrt{\sum_{k=1}^{K} (z_{ik} - z_{jk})^2}. \tag{2}$$

However, other distance models are possible. For example, Hoff, Raftery, and Handcock (2002) also include a projection model that maps actor locations to coordinates on a unit circle, and Schweinberger and Snijders (2003) extend the approach to use ultrametric distances and a hierarchical structure. Finally, the model can also include a set of (optional) covariates and associated coefficients, specified here as $\beta^T \mathbf{x}_{ij}$.[22]

[20] In their original study, Hoff, Raftery, and Handcock (2002) analyze the Florentine marriage data of Padgett and Ansell (1993), which records relations between major Florentine families during the 15th century. In that dataset, a tie between families is recorded if there is a marriage between them. In using the latent space approach to modeling network interdependencies, the authors are saying that marriage ties between families are representative of their positions in some latent social space.

[21] In the latent space model, the social space may be somewhat less well defined than in ideal point models, since the location of bills in the ideological space is often also estimated. In the latent space model, we only see the connections between legislators, which can be affected by factors other than ideology.

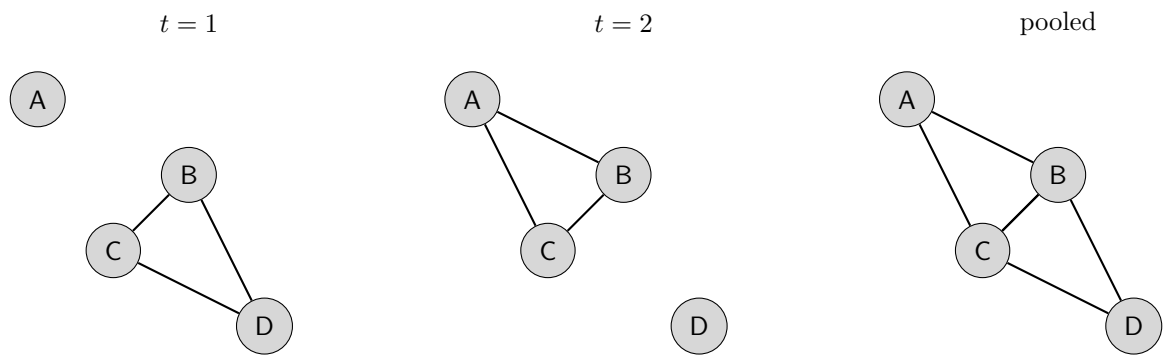
[22] In specifying covariates, it has to be remembered that the dependent variable in these models is a tie between two nodes. Consequently, covariates are often defined on the dyad.

In their original formulation, Hoff, Raftery, and Handcock (2002) demonstrated the latent space model on networks with binary ties, both directed and undirected. In that case, a logistic regression model was used. However, the latent space approach is much more general and can readily accommodate networks with more complex tie structures. Krivitsky et al. (2009), for example, demonstrate a count model that assumes a Poisson data generating process, using it to assess shared periodical readerships in Slovenia. Generally speaking, the latent space model can be easily incorporated into the generalized linear modeling framework, though to date there has been relatively little research into how well these models perform in modeling real-world data. Furthermore, several extensions to the model have been developed. For instance, Handcock, Raftery, and Tantrum (2007) extend the latent space model to include actor-level clustering, making it possible to identify groups of similar actors based only on their ties. In more recent work, Krivitsky et al. (2009) specify models to include so-called sociality random effects terms in undirected networks and sender and receiver random effects in directed networks.[23] Such terms are meant to capture the tendency for some actors to form ties more readily than others (i.e., some are more sociable than others). Finally, Hoff (2005) provides a bilinear mixed-effects model that includes the cross-product of latent sender and receiver positions. The flexibility of the latent space approach set it apart from other network methods, such as ERGMs, which have only recently been extended to value-edged networks and are computationally more demanding (Krivitsky 2012; Desmarais and Cranmer 2012).

Something should be said about the fundamental assumption underlying this model. The latent space model carries with it a strong conditional independence assumption; i.e., ties are assumed to be independent given the node positions in the latent space and any covariates included in the model. In other words, the latent positions (along with the covariates) fully capture the complex dependencies that affect tie formation in the network. However, while stringent, this is no different than the assumptions made in traditional regression models, where assumptions of conditional independence are also made. That said, unlike standard models, little is known about the sensitivity of the latent space model to deviations from the conditional independence assumption or on their performance in small networks.

[23] Krivitsky et al. (2009) also include clustering in their random effects latent space model.

Figure 3: Pooling a Dynamic Network.



3.3 The Problem with Dynamic Networks

The latent space model was originally formulated for single realizations of a network, which limits its applicability to dynamic networks. As originally conceived, scholars had two options for modeling dynamic networks with the latent space approach: they could pool all observations into a single network or they could analyze each network separately. Neither of these options is particularly satisfactory. As with other types of data, pooling a dynamic network into a single realization means masking potentially interesting processes that drive structural change. Furthermore, when networks are pooled, structures may appear different than they are. Depending on the research question being explored, the inferences we make by pooling a dynamic network may be quite misleading.

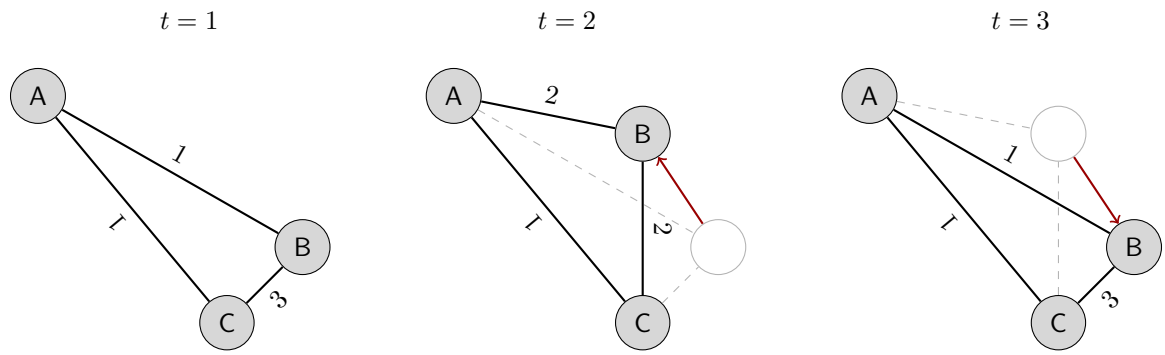
Consider the simple network depicted in Figure 3. This network consists of four nodes and two periods. In the first period, there is a transitive relationship between nodes B, C, and D. While in the second period the ties between B and D and between C and D have broken down, while at the same time, new ties were formed between A and B and between A and C. If we pool these network realizations into a single network suitable for analysis with the standard latent space model (as depicted in the third network in the figure) the structural change in the network between these two periods is no longer apparent. Instead, the network appears to be nearly fully connected, only lacking a tie between A and D.[24] Depending on the question being asked, inferences drawn from this pooled network may be wrong. For example, suppose the network represented military alliances. In this case, the

[24] In this simple network, B and C would also have identical positions in the latent space.

first and second networks represent very different worlds. In $t = 1$, country A was excluded from alliances with the other countries, suggesting it may play the role outside the system of international security or could even be a common enemy of the other countries during this period. As such, the latent position of A would be located far from the other nodes. In $t = 2$, D is the outlier, while A has now been incorporated into the alliance network of B and C. Obviously, these are very different scenarios, which would be erased in an analysis that pooled the networks together.

The second option available to scholars wanting to apply the latent space model to dynamic networks would be to model each network realization separately. Doing so would make change in network structures more apparent; however, estimating separate models raises its own problems. Consider the network of three nodes and three periods in Figure 4. In this network, ties are valued, which we may interpret as being the number of interactions between nodes during each period. In the first period, $t = 1$, B and C report three interactions with each other, while both B and C report one interaction each with A. The latent space model would, in this case, place B and C close to each other in the latent social space and far from A. In $t = 2$, the number of interactions between A and B increases to two and the number of interactions between B and C decreases to two. For this reason, the latent space model, knowing nothing about the positions of the actors in the previous period, would shift the latent positions so that the distance between A and B and between B and C were equal. Finally, in the last period, $t = 3$, the number of interactions between the nodes returns to the values observed in $t = 1$; thus, the latent positions estimated for each node by the latent space model will also revert to those of the first model. Taken as individual networks, it makes sense that the position of B would change over time; however, when looked at as a dynamic network, the relatively large changes in the position of B seems less appropriate. First, estimating three separate models ignores the knowledge about previous periods, making the estimate inefficient. For instance, given T realizations of a network with n nodes, a k-dimensional latent space model, estimated for each realization of the network, would require $T \times n \times k$ estimated parameters. However, by putting some structure on actors' movement in the social space over time, it may be possible to greatly limit the number of parameters that need to be estimated. In the example of Figure 4, $3 \times 3 \times 2 = 18$ parameters

Figure 4: Analyzing a Dynamic Network Separately.



would need to be estimated in three latent space models, while a linear trajectory model (introduced in Section 4) requires $3 \times 4 = 12$ parameters.[25] Second, estimating three separate models introduces a risk of overfitting or inferring more change in node location in the latent social space what may be the case. Figure 4 shows this potential quite clearly. As discussed above, the change in position of B in the second period looks too extreme given the observed ties in the first and third periods. If ties are a stochastic process, the pattern of ties seen in the second period would not be unusual even if the pattern in the first and third periods was the expected one. Finally, estimating separate models could make it difficult to compare latent positions across observed networks. As discussed below, model identification can be an obstacle in these models. Depending on how it is achieved, the latent spaces could be on different, incomparable scales across model estimates. This is would be particularly the case if there was node turnover in the networks, which could greatly affect node positioning.

4 The Latent Path Model

Here I introduce a new model suitable for modeling dynamic networks, such as the party switching network analyzed in the next section. The proposed latent path model builds on the latent space model of Hoff, Raftery, and Handcock (2002) outlines in the previous section and is closely related to other dynamic latent space models (Sarkar and Moore 2005; Sewell and Chen 2015; Ward, Ahlquist, and Rozenas 2013), though there are important differences

[25] The relative efficiency of the proposed model increase as the number of times periods increase.

that will be discussed below.[26] This model has a few key features. First, instead of assuming that each actor is located at a single point in the latent social space, the model treats the location of each actor as lying on a *path* in that space. In other words, actor positions are allowed to shift over observed network realizations, and the direction and magnitude of these changes is inferred from the dynamic evolution of ties in the network. Second, by explicitly linking the observed realizations of a network through the estimation of trajectories for each actor in any of the observed networks, the model provides a natural way to accommodate changes in the nodal composition of the network over time.[27] Missing data, either in the form of unaccounted for ties or missing or changing composition of modes, is a significant problem in the statistical study of networks (Robins, Pattison, and Woolcock 2004; Kossinets 2006; Borgatti, Carley, and Krackhardt 2006; Huisman and Steglich 2008). Yet, in social networks, it is common for there to be significant change in the composition of the network. For example, in the application to party switching in the Polish parliament, a great deal of MP turnover in parliament occurs with each term. If change over time is of interest, this makes existing models inappropriate. The ability of the latent path model to accommodate networks that experience some level of turnover in nodes should make it useful for analyzing a broader range of networks.

The formal definition of the latent path model is directly analogous to the latent space model of Hoff, Raftery, and Handcock (2002) described above. The difference is that the response vector in the latent path model includes repeated observations for each dyad for each time period, while the distances between actors in the network are also allowed to change over time as actors' positions in the latent social space change. Mathematically, this suggests the addition of time subscripts as well as a redefinition of the position vectors from those of Eq. (1):

$$y_{tij} = \mathcal{F}\{\beta^T \mathbf{x}_{tij} - d(\mathbf{z}_{ti}, \mathbf{z}_{tj})\}. \tag{3}$$

[26] In their conclusion, Hoff, Raftery, and Handcock (2002) mention the possibility of extending the model to dynamic networks, though they do not provide detailed guidance on doing so.

[27] Greater levels of node turnover will, at a minimum, increase the uncertainty in the estimates of the estimated latent trajectories for all nodes in the network. This is similar to a situation in item response models, where missing responses increase the standard errors around the estimates of latent abilities.

Here, the subscripts i and j continue to refer to the two nodes in a dyad in the network, while $t = 1, 2, \ldots, T$ indicates the observed realizations of the network of interest. The noticeable difference between the model of Eq. (3) and the latent space model of Eq. (1) lies in the definition of the latent positions, $\mathbf{z}_{ti}$. In the latent path model, $\mathbf{z}_{ti}$ no longer represents a single point in the k-dimensional social space; rather, it represents a path defining a trajectory of movement for actor i through the latent social space over a set of temporally-ordered network realizations.[28] For example, the simplest, non-trivial function would assume that actors in the network change positions in the latent social space following a linear trajectory (i.e., nodes are modeled as moving from point A to point B in T equivalent steps). In this case, $\mathbf{z}_{ti}$ can be defined as follows:

$$\mathbf{z}_{ti} = g(t,\, \mathbf{z}_i^0,\, \mathbf{z}_i^s) = \mathbf{z}_i^0 + t\,\mathbf{z}_i^s. \tag{4}$$

Here, $\mathbf{z}_i^0$ is a vector representing the starting positions for actor i in the latent social space and $\mathbf{z}_i^s$ is a step vector indicating the direction and magnitude of movement taken by the actors in each period. Of course, more elaborate functions are possible. One can imagine, for instance, a quadratic or otherwise curved path being specified. Alternatively, as a smaller innovation on the linear trajectory, a path function could be specified that allowed an increasing or decreasing degree of movement during each period. Such a model is used in the analysis of party switching presented in Section 5.

4.1 Identification and Estimation

Parameter identification is a significant obstacle one faces when attempting to estimate latent variable models. The lack of identification in these models takes two forms.[29] The first issue lies in the invariance of the likelihood to reflection, rotation, and translation of the latent positions. In other words, while maximizing the likelihood with respect to *distances* is possi-

[28] It is not necessary for the network to be observed at regular points in time; however, if the period of time between observations does vary, changes to the definition of the trajectory function may need to be made to account for this, depending on how important the gap between realizations is to the substantive interpretation of the problem at hand.

[29] The issues of identification described here are analogous to that faced by scholars wanting to estimate ideal-point or other latent variable models (Clinton, Jackman, and Rivers 2004; Bafumi et al. 2005).

ble and relatively straightforward (Hoff, Raftery, and Handcock 2002, p. 1092), maximizing with respect to the locations is not. For example, suppose $\mathbf{Z}$ is a matrix of n latent locations in a k-dimensional space. Consider these the true positions of these nodes. There then exists a $k \times k$ transformation matrix, $\mathbf{T}$, such that $\hat{\mathbf{Z}} = \mathbf{ZT}$ and $\mathcal{L} = \mathcal{F}(d(\hat{\mathbf{Z}})) = \mathcal{F}(d(\mathbf{Z}))$, where $d()$ is the distance function and $\mathcal{L}$ is the likelihood given the inputs. Consequently, the model is not identified with respect to node locations: there exist an infinite number of sets of locations in the latent space that map to the same set of distances between nodes.

The second source of non-identification is due to additive aliasing. Suppose $\mathbf{d} = d(\mathbf{z}_i, \mathbf{z}_j)$ $\forall\, i \neq j$, a vector of distances between each pair of nodes in the network, $\mathbf{b} = \beta\mathbf{1}$ is an intercept term (perhaps representing the overall, default connectivity of the network), and the likelihood can be seen as some function of their difference: $\mathcal{L} = \mathcal{F}(\mathbf{b} - \mathbf{d})$. In this case, one can add some value δ to both $\mathbf{b}$ and $\mathbf{d}$ and recover the same likelihood:

$$\mathcal{L} = \mathcal{F}(\mathbf{b} - \mathbf{d}) = \mathcal{F}\left((\mathbf{b} + \delta) - (\mathbf{d} + \delta)\right). \tag{5}$$

There are two approaches to dealing with the problem of identification with respect to the invariance of distances to latent locations. The first approach is to first define some particular set of latent positions, $\bar{\mathbf{Z}}$, as a reference class,[30] and then during each draw from the posterior (in a Bayesian context) use a Procrustes transformation to minimize the difference between the posterior draw and the latent positions.[31] This is the method used by Hoff, Raftery, and Handcock (2002), and subsequent studies have used a similar approach (e.g., Krivitsky et al. 2009; Shortreed, Handcock, and Hoff 2006). The second approach to modeling identification is to fix the locations of a small set of reference nodes in the latent social space (as in ideal point models, $k+1$ nodes need to be fixed to assure identification; see Clinton, Jackman, and Rivers 2004). Similarly, in a Bayesian approach, strong priors can be used to constrain the latent locations of some of the nodes to assure convergence to a single mode of the posterior.

Each of these approaches carries with it technical and substantive advantages and disadvantages. In the first approach, the use of a Procrustes transform should greatly decrease the computational burden of estimating the model, since every draw of the latent locations

[30] How these positions may be chosen will be discussed below.

[31] See Borg and Groenen (2005, ch. 20) for a description of the Procrustes transform.

from the posterior is forced, via the transform, to reflect a single mode of the posterior. In contrast, using fixed reference nodes for identification will likely be more computationally demanding, since the model will take some time to converge to the model posterior (i.e., a longer burn-in period will be required compared to the Procrustes transform approach). Finally, by relying on strong priors, the fully Bayesian approach is likely to be even more computationally onerous.

On the other hand, since the Procrustes method of transforming the latent locations used in these analyses relies on a procedure that minimizes the difference between the reference class, $\bar{\mathbf{Z}}$, and the transformed positions, $\mathbf{ZT}$, there is some risk that the draws from the posterior using this method will not reflect the variability in the actual latent locations. This is because the Procrustes transform forces each draw to be as close as possible to the reference configuration. Instead, fixing a set of reference nodes and estimating the other nodes' latent locations directly, without such a transform, should result in the posterior marginal variances for the nodes that accurately reflects the information contained in the network.

A second advantage of the fixed-node method is that it allows the analyst to "bake-in" an orientation of the latent space that is consistent with theory. Scholars often know, for instance, that some parties or politicians are to the political "left" or "right" of others; thus, it makes sense to orient estimated positions in the latent social space to reflect this prior knowledge. Fixing particular, influential nodes provides a straightforward substantive interpretation of model results without the need for post-processing.

The problem of additive aliasing can be addressed, in the Bayesian context, by using tight priors on the distances and the intercept term. In a maximum likelihood approach, aliasing is a bit more difficult to deal with; however, it is possible to post-process the positions and intercept after maximization. This is necessary even when you fix the locations of some nodes, since the simultaneous estimation of the intercept and the locations tends to increase the distances between nodes (also observed by Shortreed, Handcock, and Hoff 2006, p. 27). Alternatively, in the case where nodes can be grouped by some characteristic (for instance, by gender in a study of adolescent friendship ties or by time period in a temporal model) a hierarchical approach to specifying the priors could be used to assure identification. If

taking a maximum likelihood approach, setting one of the intercept terms could be set to zero (Bafumi et al. 2005, pp. 173–174).

Because of the inherent dependencies in network data, the large number of parameters that need to be estimated, as well as the identification issues, establishing good starting values for any of the optimization routines can greatly reduce the computational burden of these models. Hoff, Raftery, and Handcock (2002) and subsequent researchers have used a multi-step approach. In the first step, geodesic distances between each pair of nodes are calculated between nodes. Second, latent starting positions in the k-dimensional space are generated using classical mulitdimensional scaling (Gower 1966). Hoff, Raftery, and Handcock (2002) then use these positions as starting values to get point estimates for the latent locations via maximum likelihood. Their subsequent MCMC algorithm then uses the maximum likelihood estimates as the reference positions for their Procrustes transform, which establishes the posterior distribution of the node locations.[32]

4.2 Comparison to other Models

Two other latent space models have been proposed for dynamic networks. Ward, Ahlquist, and Rozenas (2013) build on the bilinear mixed effects model of Hoff (2005) to model dynamic international trade networks. That model accounts for the third-order dependencies of network data through the inclusion of the cross-products of sender and receiver positions in the latent space. When a dyad's sender and receiver positions are oriented in the same direction, this is an indication that the ties between these actors is stronger than one would expect given the rest of the model. Ward, Ahlquist, and Rozenas (2013) extend the model to include lagged bilinear terms. These terms are only included for $t = 2, 3, \ldots, T$, with the bilinear terms for all periods after the first being interpreted as changes in latent positions in the sender and receiver spaces.[33] For modeling directional network data, the bilinear model has some advantages, particularly in cases where latent sender and receiver spaces have clear

[32]The standard errors derived from maximizing the likelihood should not be used for inference given the non-independence of the data.

[33]Note that it is not possible to drop a period from the model in order to construct the lagged bilinear effect; as a latent trait of the system, there is no way to construct the lag without first estimating the positions.

theoretical interpretations. This was the case in the authors' application to international trade networks. However, for modeling affiliation or other undirected networks, the model is less useful: in these networks, there is no concept of sender and receiver. Furthermore, their model also draws the latent sender and receiver locations from different spaces; thus, if there is an interest in the location of the nodes in the latent space, this makes comparison difficult. Finally, their lag term is on the cross-product, so unit movements are not explicitly modeled.

An earlier approach to modeling dynamic networks was proposed by Sarkar and Moore (2005). One of the biggest differences between this model and the one I propose is that in their model, change in latent position is not modeled explicitly. Instead, nodes are simply allowed to drift in the latent social space. Another difference is how they model ties. Each node in the network is assumed to have a level of sociality. This sociality defines a space within which nodes are to have a decreasing likelihood of a tie with another node within the space. The probability of a tie with a node outside this circle is defined to be small. Finally, the model was constructed for binary ties and it is not clear how easily it can be extended to weighted edges. An advantage of their model, however, is that it can readily accommodate large networks, as they show in their Monte Carlo analysis, which includes networks with up to 11,000 nodes. Building on Sarkar and Moore (2005), Sewell and Chen (2015) have recently introduced a random walk model incorporating an attractions parameter meant to model sociality of individual nodes.

Random walk models are attractive in their simplicity; however, for the purpose of the substantive analysis of politics, they are quite susceptible to overfitting. For instance, in the recent model by Sewell and Chen (2015), each node noticeably shifts during each period depending on whatever ties they have during that period. If a large number of periods are included in the model, then these idiosyncratic shifts in latent position should average out. However, when only a few realizations of the network are available, these models could be influenced to a great degree by noise in the data.

5 An Analysis of Party Switching in the Polish Sejm

The latent path model presented in the previous sections provides a novel approach to assessing specific patterns of party switching and their unique effects upon party system institutionalization. Three features of the model make it particularly well suited to the task. First, the latent path model is dynamic. Thus, it is appropriate for evaluating the level of change in the party system over time. Second, the latent path model is explicitly relational. An MP's decision to switch parties is likely to affect other members' decision. Perhaps it signals the weakness of party discipline or the non-viability of the original party. Because MPs' decisions to switch are not independent, standard statistical models that assume independence of observations are not appropriate. Finally, the latent path model is spatial, which fits well with spatial theories of politics (e.g., Downs 1957).

In this section, I apply the latent path model to party switching in the Polish Sejm with the intent of assessing whether there is any connection between switching and ideology in Polish politics. Ideology is an important factor in determining the nature of the party system, providing structure to political competition and placing important constraints on elected elites (Mainwaring and Scully 1995; Mainwaring 1999; Tavits 2005, p. 286). In young, post-communist democracies, the ideological contours of politics are often unclear. Parties and politicians often fail to take clear positions on issues, and focus instead on populist appeals or personal charisma to obtain office (Innes 2002). This is certainly the case with Poland. But ideology remains an important determinant of party systems and, if institutionalization is occurring, we should see greater ideological coherence in parties over time. Here I use the proposed latent path model and the network of party switching in Poland to show that party switching has allowed Polish MPs to sort into more coherent ideological groups. In other words, contrary to conventional wisdom about the destructive consequences of party switching, the evidence presented here suggests switching can play a positive role in the process of party system institutionalization in new democracies.

Table 3: Distribution of Shared Parties by Time Period.

	# of shared parties				
Term	4	3	2	1	0
1991	0	3	10	1256	5127
1993	1	1	3	4408	6912
1997	0	1	92	5482	11261
2001	0	0	302	3352	6642
2005	0	0	1	605	1950

5.1 The Data

The network of party switching introduced in Section 2.1 were used in this analysis. Because there was a great deal of attrition of in the network—of the 1603 MPs that served in the Sejm during the five terms analyzed here, 1096 (68%) served only one term, while 308 (19%) served two terms—only those 199 MPs that served three or more terms were used in the analysis.[34] The network was also converted from a bipartite network of party affiliations to a unimodel network recording the number of shared parties between each MP in the network. The distribution of the number of shared parties between the 199 MPs included in the network is shown in Table 3.

5.2 Model Specification

Before applying the latent path model to the Polish data, several decisions about model specification need to be made. First, the number of dimensions for the latent social space has to be determined. In the absence of prior knowledge about this space, the analyst would need to estimate multiple models with different dimensions and compare them to determine what produces the best fit; for example, by using some out of sample predictive diagnostic. In the case of the Polish Sejm, however, we are able to lean heavily on prior research and knowledge of Polish politics. While there has been some debate in the literature on Polish politics about the salience of particular dimensions at particular times, area specialists are largely

[34] While the latent path model is able to accommodate attrition from and addition to the networks, estimation of the model with so much volatility in the network would not be feasible.

in agreement that Polish politics is structured around two primary dimensions of political contestation: a redistributive/pro-market economic dimension and a secularist/confessional dimension (e.g., see Markowski 2008). Thus, I have chosen to use a two dimensional latent social space.[35]

Second, the form of the trajectory has to be specified. For this analysis, I have chosen a linear trajectory (see Equation 4) for MP movement in the latent social space, with one slight modification. If party politics in the Polish parliament are settling down—party system maturation is occurring—this would suggest that MPs would be shifting their positions in the latent social space more slowly over time. For this reason, I have included in the model a decay parameter on the distance MPs move during each term in parliament. If this decay parameter is estimated to be less than 1, this would suggest that, on average, MPs are moving more slowly over time. On the other hand, if the parameter is greater than 1, this would imply that the Polish party system is increasingly chaotic.

The assumption that MPs follow a linear path through the latent social space potentially a more problematic assumption than assuming a two-dimensional latent social space. It is quite possible, for instance, that political learning by different politicians may be non-linear. That said, the linearity assumption is justified by the limitations of the available data. The Polish party switching data contains just five periods; consequently, it is likely that successfully estimating anything but a linear model would be difficult.[36] Furthermore, a more complicated specification could be susceptible to overfitting.

As described above, the unimodal network analyzed here records the number of shared parties between each MP. As a discrete, count variable, a Poisson model was used. Given the trajectory discussed above, the model is shown here:

$$y_{tij} = \text{Pois}\{\beta_t - d(\mathbf{z}_{ti}, \mathbf{z}_{tj})\} \quad \forall\, i \neq j \quad \text{where} \quad \mathbf{z}_{ti} = \mathbf{z}_i^0 + t^\alpha \mathbf{z}_i^s. \tag{6}$$

[35] A one-dimensional model was also attempted; however, it never reached convergence (even when run for twice the number of iterations as the 2-dimensional model), which is a strong indication that restricting the model to a single dimension was not reflective of Polish party politics in the Sejm.

[36] Recall that a separate trajectory is estimated for each MP included in the data. In this case, 196 different trajectories, or $196 \times 2 = 392$ trajectory parameters, are estimated for the Polish data (the locations of the reference units are treated as fixed). With just five realizations of the network available, a more complicated functional form would ask too much of the data.

In Equation (6), α is the decay parameter meant to capture the maturation of the party system.

5.3 Parameter Identification

To facilitate model estimation and identification, restrictions on the latent positions and trajectories of three MPs on the scale of the latent space was necessary.[37] From a technical standpoint, restrictions could be placed on any three MPs in the dataset; however, constraining particular MPs makes interpretation of the results more straightforward. For this analysis, I have chosen to restrict the positions of Jacek Piechota, Bronisław Komorowski, and Waldemar Pawlak. Two factors make these good candidates for restriction. First, each of these MPs is, or has been, a leader of important factions in Polish politics. Jacek Piechota of the Democratic Left Alliance (SLD), successor to the communist-era ruling party, has been a long-time left-wing politician, served as Minister of the Economy in two governments, and was a member of the ruling communist party. Bronisław Komorowski, currently serving as the President of Poland, was a prominent member of Civic Platform (PO), currently the majority party in the governing coalition, before he left the party in 2010 when he was elected President. Komorowski was also a member of Solidarity and is a good representative of the political center-right. Finally, Waldemar Pawlak has been a long-time member of the Polish Peasants Party (PSL) and was also a member of United Peoples' Party (ZSL), the agrarian satellite party during communism. He has twice been Prime Minister and served as Deputy Prime Minister during first PO/PSL coalition (2007–2011). The second, practical factor that makes Piechota, Komorowski, and Pawlak good candidates as reference units is that each of the three served in the Sejm for all five terms analyzed here. Since the locations of all other politicians in the latent social space are positioned relative to these three reference nodes, having them present in each of the time periods reduces the computational burden of estimating the model.

Choosing politicians for whom their ideological leanings are well known lends interpre-

[37] As discussed in Section 4.1, pairwise distances are invariant to reflection, rotation, and translation. Without these restrictions, it would not be possible to statistically identify unique positions for the MPs in the latent social space. A similar restriction is required for identification in ideal point models (see, e.g., Clinton, Jackman, and Rivers 2004; Bafumi et al. 2005).

Table 4: Box Constrains on the Latent Locations for the Reference MPs.

	Economic	Social
Piechota	$[-5, 0]$	$[-5, 0]$
Komorowski	$[0, 5]$	$[-5, 0]$
Pawlak	$[-5, 5]$	$[0, 5]$

tive structure to the latent social space. In this case, relative latent locations can be chosen for each of these MPs in a way that agrees with prior knowledge about their political leanings. Specifically, as a leader of the ex-communist party, Piechota is known to be left of Komorowski on economic issues, while also being less socially conservative. Between Komorowski and Pawlak, we know that Komorowski was less socially conservative than Pawlak and also somewhat to the right on economic issues. Thus, by positioning these politicians in a way that reflects this knowledge, the estimated latent positions of the other MPs become interpretable on these ideological dimensions. However, instead of assigning Piechota, Komorowski, and Pawlak arbitrary locations that satisfy their relative positions in the social space, which would have guaranteed model identification, their locations in the space were estimated simultaneously with the locations and trajectories of all other MPs, subject to some strong priors and box constraints on their latent locations. These box constraints are shown in Table 4. In addition, a normal prior with mean of zero and standard deviation of 0.25 was placed on their locations. This is a conservative specification, since it means any distance away from the origin would be a strong sign that the network contains information about differences in their political leanings.

Priors for all other parameters estimated are reported in Table 5. As shown, all parameters were given relatively tight priors to facilitate identification. However, as with the locations of the reference MPs, this is a conservative specification, since any estimated differences in latent locations would indicate clear differences in ideological positioning. On the other hand, if MPs' estimated positions are clustered around the origin, this would be an indication that the network carries with it very little information.

Table 5: Priors for Intercepts, Decay Parameter, Latent locations, and Trajectories for all other Parameters in the Latent Path Model.

Parameter	Prior
Intercept (β_{0t})	Normal($\mu = 0$, $\sigma = 1$)
Decay† (α)	Normal($\mu = 1$, $\sigma = 1$)
Positions ($\mathbf{z}_i^0$)	Normal($\mu = 0$, $\sigma = 1$)
Trajectory ($\mathbf{z}_i^s$)	Normal($\mu = 0$, $\sigma = 0.5$)

† decay parameter was constrained to be positive.

5.4 Model Estimation

The model was estimated using Hamiltonian Monte Carlo in Stan (Stan Development Team 2013) using the No-U-Turn sampler (Hoffman and Gelman 2013). Four chains were simulated with 400,500 iterations each, saving the last 500 draws from the posterior for each chain. Gelman-Rubin's (1992) $\hat{R}$ was less than 1.05 for these as well. Summary of model convergence is available in Appendix B. Stan code for the model is located in Appendix C.[38]

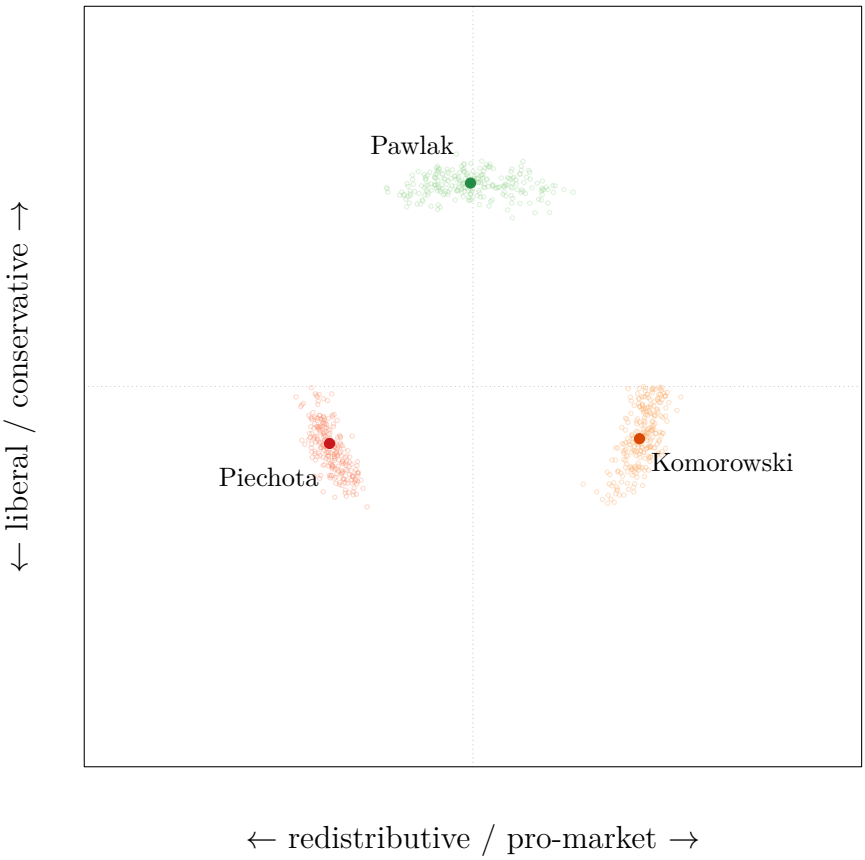
5.5 Results

The locations of the reference MPs—Piechota, Komorowski, and Pawlak—is key to the interpretation of the model results. Figure 5 shows a random draw of 250 points from the posterior distribution of estimated final locations for these MPs. The large, solid points represent the posterior means. As expected, the model positions Piechota, Komorowski, and Pawlak in the lower-left, upper-left, and lower-right quadrants, respectively. This is, of course, a result of the relative constraints placed on their positions in the model. What is noteworthy, however, is the distances of the posterior means from the origin. Recall that the model does permit (and even encourages given the tight normal prior on their positions) the positions of these reference units to collapse to the origin, which would happen if the

[38] Convergence was not perfect. Of the 797 parameters estimated (includes the log-probability), 93 (11.6%) $\hat{R}$ values were over 1.05 (the usual threshold accepted as indicating convergence); however, only 5 parameters had an $\hat{R}$ above 1.10. It is not expected that this lack of apparent convergence should change the results presented below as it appears that three of the four chains had converged, but that the fourth was more problematic. A greater number of iterations should solve this.

party switching network contained no information about the ideological locations of these actors. Instead, structural patterns in the switching network clearly indicate ideological differentiation.

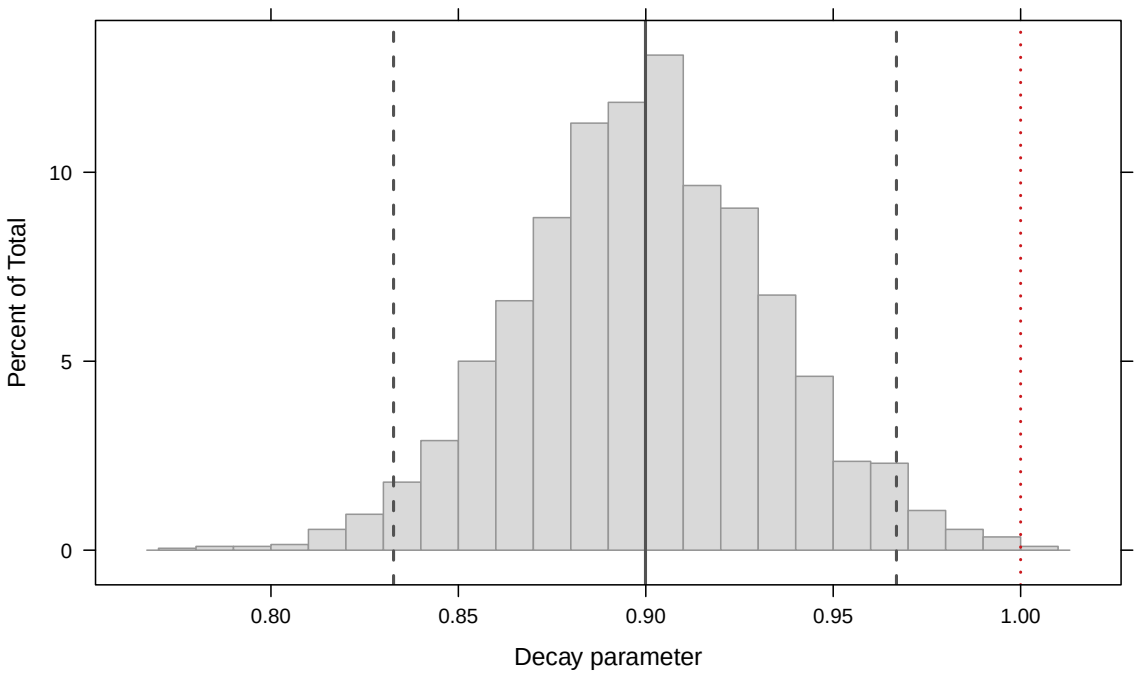
Figure 5: Estimated latent positions for reference MPs, Piechota (red), Komorowski (orange), and Pawlak (green). Open points are 250 random draws from the posterior positions for each MP. Means for the full marginal posteriors are shown as dark, closed points.



It was argued above that if the Polish party system is settling into a more stable equilibrium, the movement of MPs in the latent social space will slow over time. Figure 6 reports the posterior distribution of the estimated decay parameter and provides evidence for this hypothesis. As shown, the posterior mean for the decay parameter is approximately 0.9. A value less than 1.0 indicates that, on average, MPs' movements through the latent space are, in fact, slowing.[39]

[39]While the slowing is not dramatic—movement between the 4th and 5th periods had decreased to just

Figure 6: Posterior Distribution of Trajectory Decay Parameter. Solid vertical line is the posterior mean, while the dashed lines define the 95% credible interval.



Finally, the full results from the estimated latent path model are presented graphically in Figure 7. This figure plots the estimated positions in the latent social space for each active MP in the Polish Sejm over the five parliamentary terms included in the study, with each point in the plot representing one of the 199 MPs included in the estimation. Since the estimated positions of the MPs in the latent social space can overlap when they have identical or similar ties, MP locations have been plotted with some transparency, so that the darker the area the larger the number of MPs located at that location. To aid interpretation, labels are also included for several groups (parties) of interest.

Several things are apparent from these results. First, despite the expected organizational strength of Solidarity and the Catholic Church at the beginning of the democratic transition, the political right appears to have been in disarray during much of this period. This is apparent in the scattered latent positions estimated from the data, which is particularly

under 80% of what it was between the first and second periods—it should be remembered that this is an average rate for all MPs in the dataset.

noticeable in the first and fourth terms. Also clear is the split between PiS and PO in 2001.

The second thing to notice is that SLD has remained fairly stable and cohesive during this period. Very few of its members have switched to or from the party, which is apparent from the lack of MPs being placed by the latent path model into positions between the SLD MPs and other groups. Even while in turmoil during the 2001 term, members did not seek out parties outside the leftist, social democratic family of parties.

Recall that the relative disorder on the right, the stability of SLD, and the growing association of PSL with the right was also reflected in the standard network plots of Figure 2. Along with qualitative knowledge of the Polish party system, this helps lend credibility to the latent path model. However, the network plots also obscured the degree to which the Polish parliament has come to be dominated by four relatively homogeneous factions. While switching clearly continues, particularly on the right, this switching has come to take place almost exclusively between more homogeneous groups over time. This is evident in the fact that very few MPs are positioned significantly far from groups of other MPs. In fact, by the 2005 parliamentary term, only a couple MPs are located in positions that could be considered between groups. Overall, these results lend support to the hypothesis that party switching, even at the levels seen in Poland, are not necessarily a long-term liability to the process of party system institutionalization and democratic consolidation. Instead, in Poland it appears that patterns of party switching reflects a process of political learning and the emergence of a more ideologically coherent party system.

Figure 7: Estimated Latent Locations as Produced by the Latent Path Model.

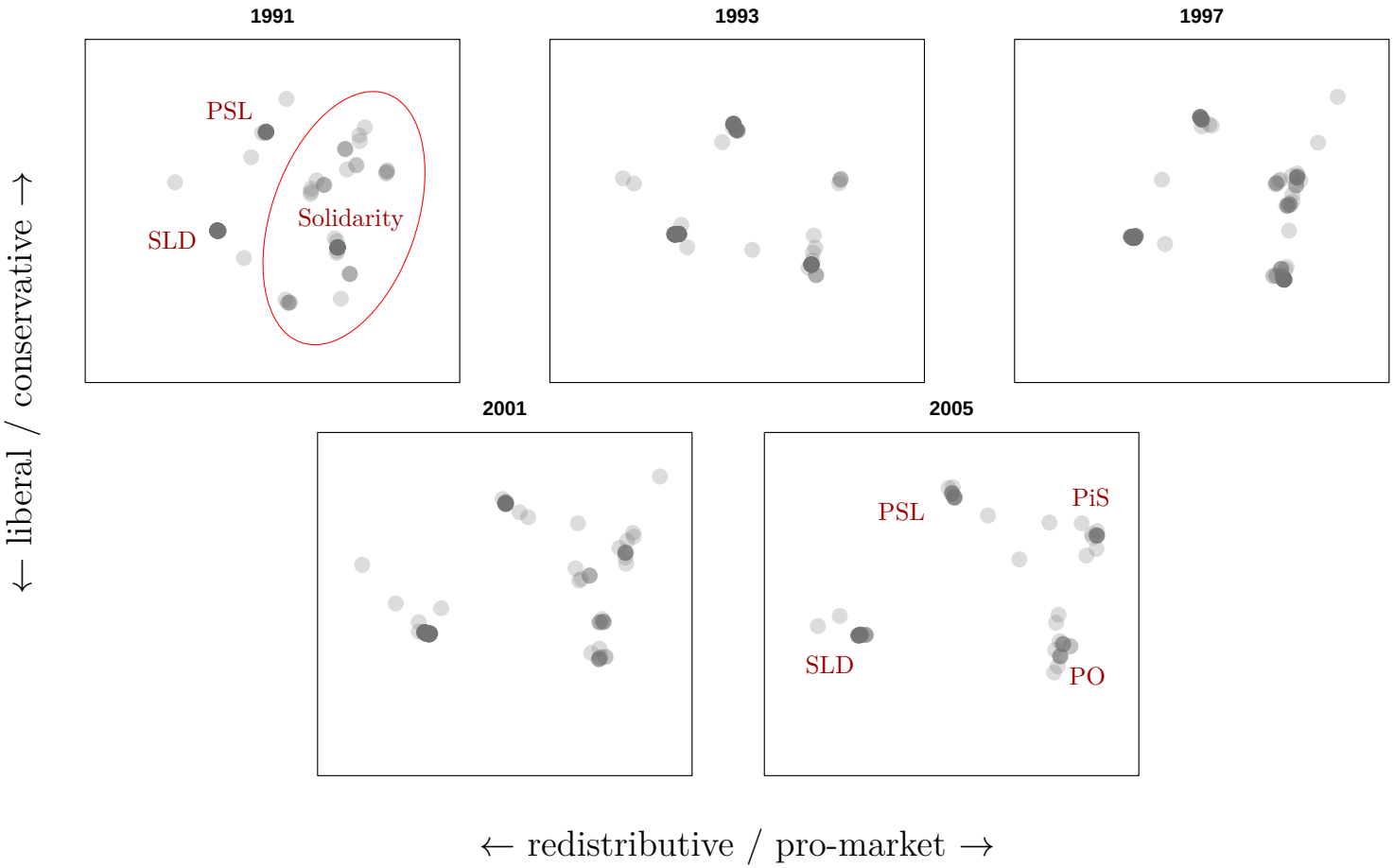
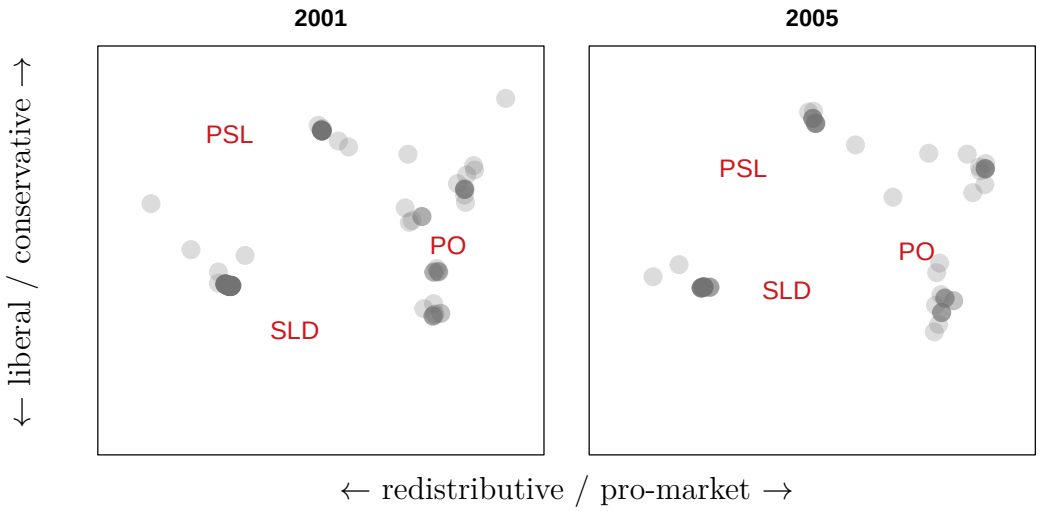


Figure 8: Comparing Estimated Latent Locations of MPs to Measures of Ideology as Provided by the Chapel Hill Expert Survey (2002, 2006).



While the estimated locations for Polish MPs presented above support the hypothesis that the Polish party system is on a path towards institutionalization, there may be some concern that the ideological labels do not conform to those actually salient in Polish politics. To validate these locations as representing ideology, Figure 8 presents a comparison of latent locations to those provided by the Chapel Hill expert survey for SLD, PiS, and PO (Hooghe et al. 2010).[40] Clearly, the ideological locations provided by the Chapel Hill survey conform closely to those generated by the latent path model. In other words, it is reasonable to conclude that the structure of party switching in the Polish Sejm is related to the ideological orientation of MPs and that the latent path model is able to recover these orientations independent of any other exogenous information, such as party manifestos or voting records.

[40]Social ideological location refers to "galton" measure; economic ideological location refers to the "lrecon" measure. To put the numbers on the appropriate scale from $[-5, 5]$, 5 was subtracted from the measures of ideology and then shifting their centroid to equal that of the reference MPs. No other transformations or rescaling was performed.

6 Conclusion

This paper has made two contributions, one methodological and one substantive. In terms of the methodological contribution, I have presented a new network model for dynamic, relational data that will be more attractive to those political scientists hoping to account for the dependencies inherent to relational data, while remaining in the more familiar the realm of GLMs. Based on the latent space approach to modeling network interdependencies (Hoff, Raftery, and Handcock 2002), this model, which I call the latent path model, supports binary or valued ties as well as directed or undirected ties. Furthermore, unlike alternative methods recently developed in the literature, the latent path model allows for the specification of non-linear movement of actors in the latent space over time.

The paper also contributes to the substantive literature on party system institutionalization in new democracies, particularly with respect to the problem of party switching. Most extant literature argues that rampant party switching is detrimental to the development of a healthy democracy. However, an application of the latent path analysis to the party switching network in Poland shows that such switching is not *necessarily* a problem. Instead, contrary to conventional wisdom, party switching in Poland has allowed politicians to sort into more homogeneous groups. For scholars of Polish party politics, this lends support to the idea that the party system in the country is finally starting to show signs of settling down (Markowski 2008). For parties scholars more generally, the results suggest we look more closely at the role of party switching in the development of stable party politics.

Appendices

A The Polish Electoral System & Election Results

Poland has a semi-presidential system, with a bicameral legislature. The Sejm, the lower house, consists of 460 MPs elected through an open list PR system. The Senate, the upper house, consists of 100 members elected by plurality (Zielinski, Słomczynski, and Shabad 2005, pp. 376–377). The center of power in the Polish system resides in the Sejm, though it the President and the Senate maintained a good deal of influence until the 1997 constitution was adopted.

A.1 Summary of Electoral Rules

Table 6: Summary of Electoral Rules for Elections to the Polish Sejm, 1991–2011.

	Year of election						
	1991⋆	1993⋆	1997	2001⋆	2005⋆	2007	2011⋆
Electoral districts[1]	37	52	52	41	41	41	41
District magnitude							
Minimum	7	3	3	7	7	7	7
Maximum	17	17	17	19	19	19	20
Median	10	7	7	11	11	11	12
Mean	10.6	7.5	7.5	11.2	11.2	11.2	11.2
Seat distribution							
Constituency	391	391	391	460	460	460	460
National list	69	69	69	0	0	0	0
Electoral threshold							
Party	None	5%	5%	5%	5%	5%	5%
Coalition	None	8%	8%	8%	8%	8%	8%
National list	5%	7%	7%	–	–	–	–
Allocation system	Hare-Niemeyer	d'Hondt	d'Hondt	Sainte-Laguë	d'Hondt	d'Hondt	d'Hondt

⋆ Indicates a change in electoral institutions. [1] Excludes the national tier. *Sources*: Gebethner (2006) and Birch et al. (2002, table 2.1, p. 27), the Polish National Electoral Commission, and author's calculations.

A.2 Polish Election Details and Results: 1989–2011

Table 7: Dates of Local, Parliamentary, and Presidential Elections and National Referenda in Poland.

Institution	Date	Winner	Notes
Parliament	1989–06–04	Solidarity	Partially contested
President	1990–11–25	Wałęsa (Solidarity)	
Local	1990–05–27		
Parliament	1991–10–27	DU (12.5%, 62 seats)	29 parties won seats in Sejm, no threshold
Parliament	1993–09–19	SLD (20.4%, 171 seats)	8 parties won seats in Sejm
Local	1994–06–19		
President	1995–11–05	Kwasniewski (SLD)	
Parliament	1997–09–11	AWS (33.8%, 201 seats)	6 parties won seats in Sejm
Local	1998–10–11		
President	2000–10–08	Kwasniewski (SLD)	12 candidates contested, Kwasniewski won in first round
Parliament	2001–09–23	SLD-UP (41.0%, 216 seats)	AWS (now known as AWSP) dissolved
Local	2002–10–27		
Referendum	2003–06–08	Vote to join EU	
Parliament	2005–09–25	PiS (27.0%, 155 seats)	7 parties won seats in Sejm
President	2005–10–09	Kaczyński (PiS)	
Local	2006–11–12		
Parliament	2007–10–21	PO (41.5%, 209 seats)	5 parties won seats in Sejm
President	2010–07–04	Komorowski (PO)	
Local	2010–11–21		
Parliament	2011–10–09	PO (39.2%, 212 seats)	6 parties won seats in Sejm

Table 8: Election Results for the Polish Sejm, 1991.

Party	Votes	Pct	Seats
Democratic Union (DU)	1382051	12.3	62
Democratic Left Alliance (SLD)	1344820	12.0	60
Catholic Election Action (WAK)	980304	08.7	49
Citizen's Centre Agreement (POC)	977344	08.7	44
Polish Peasant Party - PA (PSL-SP)	972952	08.7	48
Confederation for an Independent Poland (KPN)	841738	07.5	46
Liberal Democratic Congress (KLD)	839978	07.5	37
Others	820108	07.3	0
Peasant Accord (PL)	613626	05.5	28
Independent Self-Governing Trade Union - Solidarity	566553	05.0	27
Polish Party of Friends of Beer (PPPP)	367106	03.0	16
Christian Democrats (CD)	265179	02.2	5
Union of Political Realists (UPR)	253024	02.2	3
Labour Solidarity (SP)	230975	02.1	4
Democratic Party (SD)	159017	01.4	1
German Minority (MN)	132059	01.2	7
Party of Christian Democrats (PCD)	125314	01.1	4
Party X (PX)	52735	00.5	3
Democratic-Social Movement (RDS)	51656	00.5	1
Peasant Election Alliance (LPW)	42031	00.4	1
Silesian Autonomy Movement (RAS)	40061	00.4	2
Krakow Coalition of Solidarity with the President (KKSP)	27586	00.2	1
Podhalan Union (ZP)	26744	00.2	1
Polish Western Union (PZZ)	26053	00.2	4
Great Poland and Poland (WPP)	23188	00.2	1
Peasant Unity (JL)	18902	00.2	1
Electoral Committee of Orthodox Believers (KWP)	13788	00.1	1
Solidarity 80 (S 80)	12769	00.1	1
Union of Great Poles (UWL)	9019	00.1	1
Alliance of Women against Life's Hardships (SKPTZ)	1922	00.0	1

Table 9: Election Results for the Polish Sejm, 1993.

Party	Votes	Pct	Seats
Alliance of the Democratic Left (SLD)	2815169	20.4	171
Others	2186799	19.5	0
Polish Peasant Party (PSL)	2124367	15.4	132
Democratic Union (UD)	1460957	10.6	74
Labour Union (UP)	1005004	07.3	41
Confederation for Independent Poland (KPN)	795487	05.8	22
Non Party Reform Bloc (BBWR)	746653	05.4	16
German Minority of Opole Silesia	60770	00.4	3
Germans of Katowice Province (NWK)	23396	00.2	1

Table 10: Election Results for the Polish Sejm, 1997.

Party	Votes	Pct	Seats
Solidarity Election Action (AWS)	4427373	33.8	201
Democratic Left Alliance (SLD)	3517866	26.9	164
Others	1808674	13.8	0
Freedom Union (UW)	1723811	13.2	60
Polish Peasant Party (PSL)	900271	06.9	27
Mvt for the Recn of Poland (ROP)	662668	05.1	6
German Social and Cultural Soc (MNO)	51027	00.4	2

Table 11: Election Results for the Polish Sejm, 2001.

Party	Votes	Pct	Seats
Democratic Left Alliance - Union of Labour (SLD-UP)	5342519	41.0	216
Citizens' Platform (PO)	1651099	12.7	65
Self-Defence (SO)	1327624	10.2	53
Law and Justice (PiS)	1236787	09.5	44
Polish People's Party (PSL)	1168659	09.0	42
League of Polish Families (LPR)	1025148	07.9	38
Solidarity Electoral Action Work (AWSP)	729207	05.6	0
Freedom Union (UW)	404074	03.1	0
Others	85582	00.6	0
German Minority (MN)	47230	00.4	2

Table 12: Election Results for the Polish Sejm, 2005.

Party	Votes	Pct	Seats
Law and Justice (PiS)	3185714	27.0	155
Citizens' Platform (PO)	2849259	24.1	133
Self-Defence of the Republic of Poland (SRP)	1347355	11.4	56
Democratic Left Alliance (SLD)	1335257	11.3	55
Others	1128334	10.7	0
League of Polish Families (LPR)	940726	08.0	34
Polish People's Party (PSL)	821656	07.0	25
German Minority (MN)	34469	00.3	2

Table 13: Election Results for the Polish Sejm, 2007.

Party	Votes	Pct	Seats
Citizens' Platform (PO)	6701010	41.5	209
Law and Justice (PiS)	5183477	32.1	166
Left and Democrats (LiD)	2122981	13.1	53
Polish People's Party (PSL)	1437638	08.9	31
Self-Defence of the Republic of Poland (SRP)	247335	01.5	0
League of Polish Families (LPR)	209171	01.3	0
Others	208128	02.3	0
German Minority (MN)	32462	00.2	1

Table 14: Election Results for the Polish Sejm, 2011.

Party	Votes	Pct	Seats
Citizens' Platform (PO)	5629773	39.2	207
Law and Justice (PiS)	4295016	29.9	157
Ruch Palikota (RP)	1439490	10.0	40
Polish People's Party (PSL)	1201628	08.4	28
Union of the Democrat Left (SLD)	1184303	08.2	27
Others	591279	04.1	0
German Minority (MN)	28014	00.2	1

B Convergence Statistics

Figure 9: Trace and Density Plots for Log-Probability (4 chains).

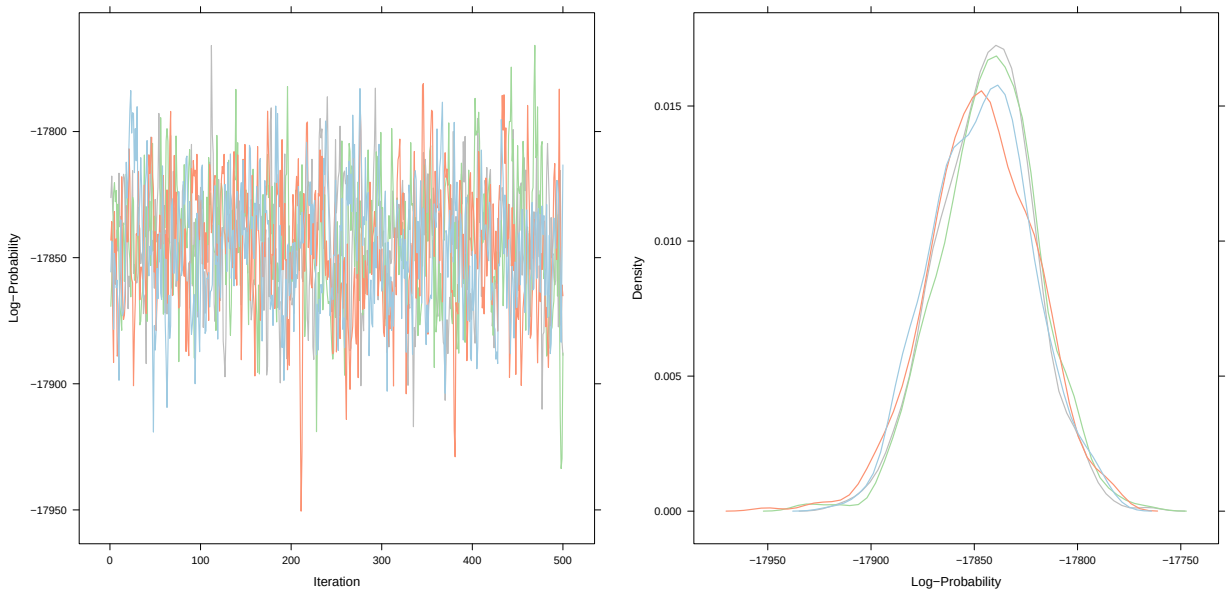


Figure 10: Trace and Density Plots for Decay Parameter (4 chains).

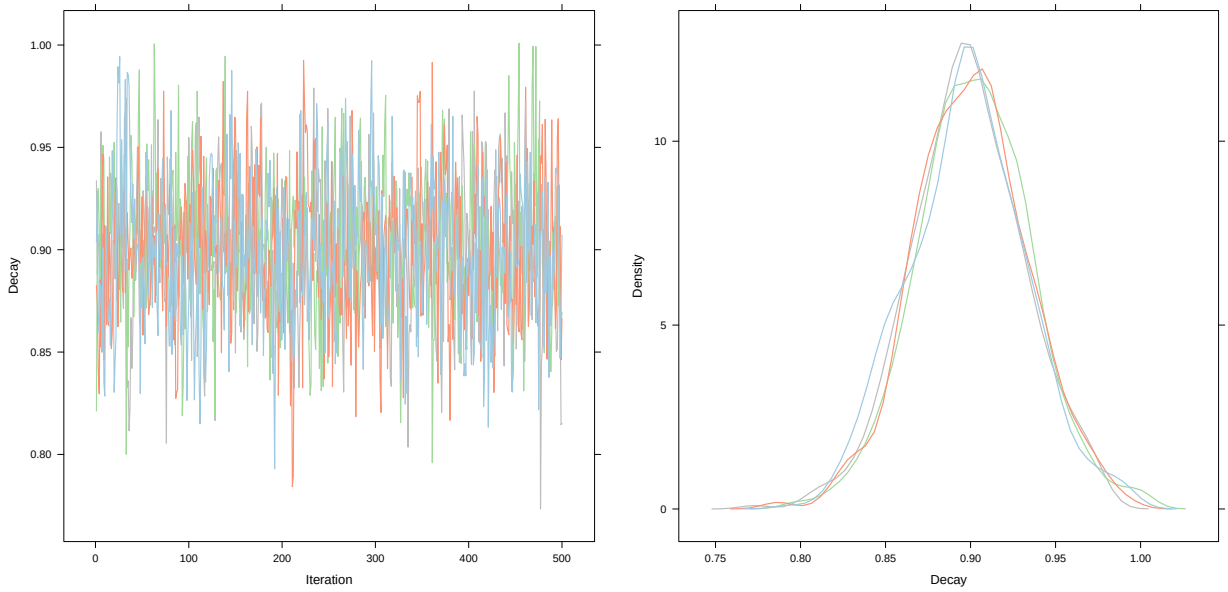
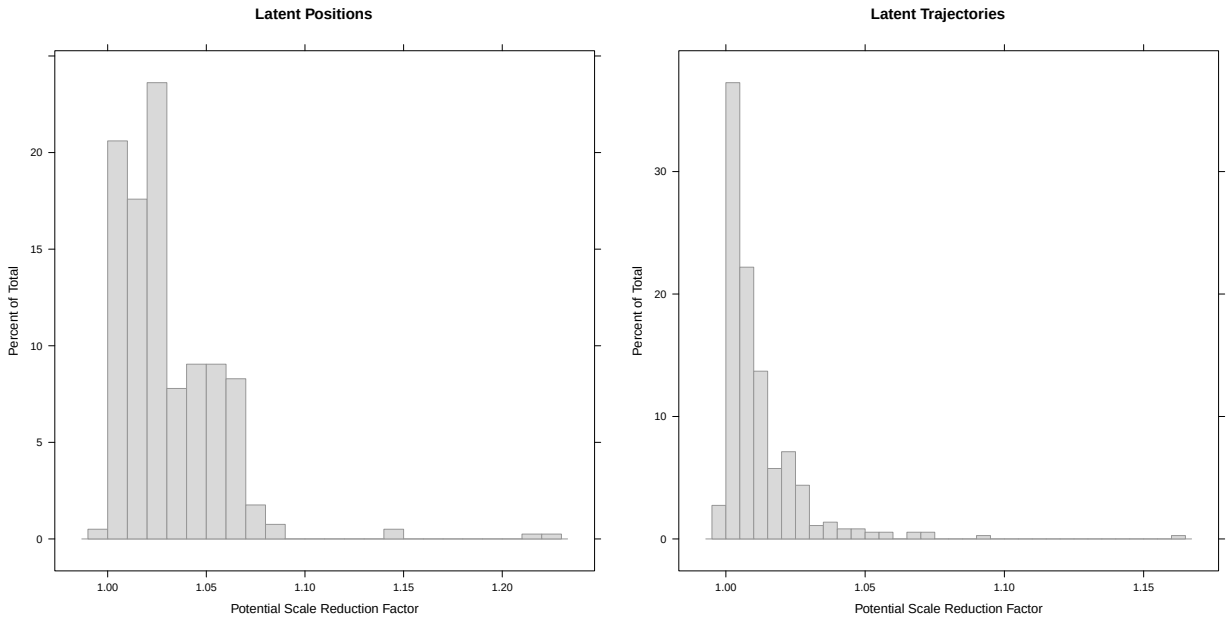


Figure 11: Distribution of the Potential Scale Reduction Factor for Estimated Latent Positions and Trajectories.



C Source Code for Stan Model

```
// ==============================================================================
// Latent Path Model:  2 dimensional
//                     Poisson
//                     fixed references
//                     decay
//
// Copyright (C) 2015: Jason W. Morgan <morgan.746@osu.edu>
// ==============================================================================

data {
  int<lower=1> n;            // total number of nodes
  int<lower=1> N;            // number of observed dyads
  int<lower=1> T;            // number of time periods
  int<lower=1> K;            // number of dimensions in latent space

  // These arrays track the outcome and the associated time period and indices
  // of the nodes
  int<lower=1> t_idx[N];     // time period of observed outcome
  int<lower=1> node1_idx[N];
  int<lower=1> node2_idx[N];
  int<lower=0> y[N];         // outcome
}
parameters {
  vector[T] beta;
  real<lower=0> decay;

  // Piechota, lower-left quadrant (III)
  real<lower=-5,upper=0> pos_x1;  real<lower=-5,upper=0> pos_y1;

  // Komorowski, lower-right quadrant (IV)
  real<lower=0,upper=5> pos_x2;   real<lower=-5,upper=0> pos_y2;

  // Pawlak,  upper-left quadrant (II)
  real<lower=-5,upper=5> pos_x3;  real<lower=0,upper=5> pos_y3;

  matrix[n-3,K] pos_raw;
  matrix[n-3,K] traj_raw;
}
model {
  matrix[n,K] pos;               // positions with constrained obs added
  matrix[n,K] traj;              // positions with constrained obs added
  matrix[n,K] positions[T];      // container for calculated positions

  vector[N] D;                   // container for beta - pairwise distances
  row_vector[2] node1_pos;
  row_vector[2] node2_pos;

  real beta_mu;
```

```
  // Priors ------------------------------------------------------------------

  beta ~ normal(0, 1);
  decay ~ normal(1, 1);

  // Centered at the origin with a tight variance so that any differences
  // are indicative of differences in latent position.
  pos_x1 ~ normal( 0, 0.25);  pos_y1 ~ normal( 0, 0.25);
  pos_x2 ~ normal( 0, 0.25);  pos_y2 ~ normal( 0, 0.25);
  pos_x3 ~ normal( 0, 0.25);  pos_y3 ~ normal( 0, 0.25);

  for (k in 1:K) {
    col(pos_raw, k) ~ normal(0, 1);
    col(traj_raw, k) ~ normal(0, 0.5);
  }

  // Containers --------------------------------------------------------------

  // Constrained nodes
  pos[1,1] <- pos_x1;  pos[1,2] <- pos_y1;
  pos[2,1] <- pos_x2;  pos[2,2] <- pos_y2;
  pos[3,1] <- pos_x3;  pos[3,2] <- pos_y3;

  // Trajectories for references are fixed at zero
  traj[1,1] <- 0;  traj[1,2] <- 0;
  traj[2,1] <- 0;  traj[2,2] <- 0;
  traj[3,1] <- 0;  traj[3,2] <- 0;

  // Unconstrained nodes
  for (i in 1:(n-3)) {
    pos[i+3,1]  <- pos_raw[i,1];
    pos[i+3,2]  <- pos_raw[i,2];
    traj[i+3,1] <- traj_raw[i,1];
    traj[i+3,2] <- traj_raw[i,2];
  }

  // Model -------------------------------------------------------------------

  // Calculate positions for each node
  positions[1] <- pos;

  for (t in 2:T) { positions[t] <- pos + pow(t-1, decay) * traj; }

  // Calculate pairwise distances for each observed dyad; adjusted for
  // time-specific intercept (beta).
  beta_mu <- mean(beta);
  for (i in 1:N) {
    node1_pos <- positions[t_idx[i]][node1_idx[i]];
    node2_pos <- positions[t_idx[i]][node2_idx[i]];
    D[i] <- beta[t_idx[i]] - distance(node1_pos, node2_pos);
  }

  y ~ poisson_log(D);
}
```

References

Bafumi, Joseph et al. (2005). "Practical Issues in Implementing and Understanding Bayesian Ideal Point Estimation". In: *Political Analysis* 13.2, pp. 171–187.

Bakke, Elisabeth and Nick Sitter (2005). "Patterns of Stability: Party Competition and Strategy in Central Europe since 1989". In: *Party Politics* 11.2, pp. 243–263.

Bielasiak, Jack (2002). "The Institutionalization of Electoral and Party Systems in Postcommunist States". In: *Comparative Politics* 34.2, pp. 189–210.

Birch, Sarah (2003). *Electoral Systems and Political Transformation in Post-Communist Europe*. New York, NY: Palgrave Macmillan.

Birch, Sarah et al. (2002). *Embodying Democracy: Electoral System Design in Post-Communist Europe*. New York: Palgrave Macmillan.

Borg, Ingwer and Patrick J. F. Groenen (2005). *Modern Multidimensional Scaling: Thoery and Applications*. Second. Springer.

Borgatti, Stephen P., Kathleen M. Carley, and David Krackhardt (2006). "On the Robustness of Centrality Measures under Conditions of Imperfect Data". In: *Social Networks* 28.2, pp. 124–136.

Casal Bértoa, Fernando (2012). "Party Systems and Cleavage Structures Revisited: A Sociological Explanation of Party System Institutionalization in East Central Europe". In: *Party Politics*.

Casal Bértoa, Fernando and Peter Mair (2012). "Party Government in the New Europe". In: ed. by Hans Keman and Ferdinand Müller-Rommel. Routledge. Chap. Party System Institutionalisation Across Time in Post-Communist Europe, pp. 85–112.

Clinton, Joshua, Simon Jackman, and Douglas Rivers (2004). "The Statistical Analysis of Roll Call Data". In: *The American Political Science Review* 98.2, pp. 355–370.

Conrad, Courtenay Ryals and Sona N. Golder (2010). "Measuring Government Duration and Stability in Central Eastern European Democracies". In: *European Journal of Political Research* 49.119–150.

Converse, Philip E. (1969). "Of Time and Partisan Stability". In: *Comparative Political Studies* 2.2, pp. 139–171.

Cranmer, Skyler J. and Bruce A. Desmarais (2011). "Inferential Network Analysis with Exponential Random Graph Models". In: *Political Analysis* 19, pp. 66–86.

Desmarais, Bruce A. and Skyler J. Cranmer (2012). "Statistical Inference for Valued-Edge Networks: The Generalized Exponential Random Graph Model". In: *PLoS ONE* 7.1.

Desposato, Scott W. (2006). "Parties for Rent? Ambition, Ideology, and Party Switching in Brazil's Chamber of Deputies". In: *American Journal of Political Science* 50.1, pp. 62–80.

— (2009). "Political Parties and Legislative Party Switching". In: ed. by William B. Heller and Carol Mershon. Palgrave Macmillan. Chap. Party Switching in Brazil: Causes, Effects, and Representation, pp. 109–144.

Downs, Anthony (1957). *An Economic Theory of Democracy*. New York: Harper & Bros.

Duch, Raymond M. (2001). "A Developmental Model of Heterogeneous Economic Voting in New Democracies". In: *The American Political Science Review* 95.4, pp. 895–910.

Epperly, Brad (2011). "Institutions and Legacies: Electoral Volatility in the Postcommunist World". In: *Comparative Political Studies* 44.7, pp. 829–853.

Fruchterman, Thomas M. J. and Edward M. Reingold (1991). "Graph Drawing by Force-Directed Placement". In: *Software: Practice and Experience* 21.11, pp. 1129–1164.

Gebethner, Stanisław (2006). "Seats and Votes: Political Consequences of the Polish Electoral System". In: *Founding Elections in Eastern Europe: The 1991 and 1993 Elections to the Polish Sejm: Analysis, Documents and Data*. Ed. by Krzysztof Jasiewicz and Radosław Markowski. Berlin, Germany: edition sigma, pp. 60–76.

Gelman, Andrew and Donald B. Rubin (1992). "Inference from Iterative Simulation Using Multiple Sequences". In: *Statistical Science* 7.4, pp. 457–472.

Gower, J. C. (1966). "Some Distance Properties of Latent Root and Vector Methods used in Multivariate Analysis". In: *Biometrika* 53.3–4, pp. 325–338.

Grotz, Florian and Till Weber (2012). "Party Systems and Government Stability in Central and Eastern Europe". In: *World Politics* 64.4, pp. 699–740.

Grzymała-Busse, Anna (2002). *Redeeming the Communist Past: The Regeneration of Communist Parties in East Central Europe*. Cambridge, UK: Cambridge University Press.

Gwiazda, Anna (2009). "Poland's Quasi-Institutionalized Party System: The Importance of Elites and Institutions". In: *Perspectives on European Politics and Society* 10.3, pp. 350–376.

Handcock, Mark S., Adrian E. Raftery, and Jeremy M. Tantrum (2007). "Model-based Clustering for Social Networks". In: *Journal of the Royal Statistical Society: Series A* 170.2, pp. 301–354.

Heller, William B. and Carol Mershon (2005). "Party Switching in the Italian Chamber of Deputies, 1996–2001". In: *The Journal of Politics* 67.2, pp. 536–559.

— (2009). "Political Parties and Legislative Party Switching". In: ed. by William B. Heller and Carol Mershon. Palgrave Macmillan. Chap. Introduction: Legislative Party Switching, Parties, and Party Systems, pp. 3–28.

Hoff, Peter D. (2005). "Bilinear Mixed Effects Models for Dyadic Data". In: *Journal of the American Statistical Association* 100.469, pp. 286–295.

Hoff, Peter D., Adrian E. Raftery, and Mark S. Handcock (2002). "Latent Space Approaches to Social Network Analysis". In: *Journal of the American Statistical Association* 97.460, pp. 1090–1098.

Hoffman, Matthew D. and Andrew Gelman (2013). "The No-U-Turn Sampler: Adaptively Setting Path Lengths in Hamiltonian Monte Carlo". In: *Journal of Machine Learning Research*.

Hooghe, Liesbet et al. (2010). "Reliability and Validity of Measuring Party Positions: The Chapel Hill Expert Surveys of 2002 and 2006". In: *European Journal of Political Research* 49, pp. 684–703.

Huisman, Mark and Christian Steglich (2008). "Treatment of Non-response in Longitudinal Network Studies". In: *Social Networks* 30.4, pp. 297–308.

Huntington, Samuel P. (1968). *Political Order in Changing Societies*. New Haven, CT: Yale University Press.

Innes, Abby (2002). "Party Competition in Postcommunist Europe: The Great Electoral Lottery". In: *Comparative Politics* 35.1, pp. 85–104.

Kadushin, Charles (2012). *Understanding Social Networks: Theories, Concepts, and Findings*. Oxford, UK: Oxford University Press.

Kossinets, Gueorgi (2006). "Effects of Missing Data in Social Networks". In: *Social Networks* 28.3, pp. 247–268.

Kostadinova, Tatiana (2003). "Voter Turnout Dynamics in Post-Communist Europe". In: *European Journal of Political Research* 42, pp. 741–759.

Kreuzer, Marcus and Vello Pettai (2003). "Patterns of Political Instability: Affiliation Patterns of Politicians and Voters in Post-Communist Estonia, Latvia, and Lithuania". In: *Studies in Comparative International Development* 38.2, pp. 76–98.

Krivitsky, Pavel N. (2012). "Exponential-family Random Graph Models for Valued Networks". In: *Electronic Journal of Statistics* 6, pp. 1100–1128.

Krivitsky, Pavel N. et al. (2009). "Representing Degree Distributions, Clustering, and Homophily in Social Networks with Latent Cluster Random Effects Models". In: *Social Networks* 31, pp. 204–213.

Laasko, Markku and Rein Taagepera (1979). "'Effective' Number of Parties: A Measure with Application to West Europe". In: *Comparative Political Studies* 12.1, pp. 3–27.

Laver, Michael and Kenneth Benoit (2003). "The Evolution of Party Systems between Elections". In: *American Journal of Political Science* 47.2, pp. 215–233.

Levendusky, Matthew (2009). *The Partisan Sort: How Liberals Became Democrats and Conservatives Became Republicans*. Chicago, IL: The University of Chicago Press.

Lewis, Paul G. (2000). *Political Parties in Post-Communist Eastern Europe*. New York, NY: Routledge.

Mainwaring, Scott (1998). "Party Systems in the Third Wave". In: *Journal of Democracy* 9.3, pp. 67–81.

— (1999). *Rethinking Party Systems in the Third Wave of Democratization: The Case of Brazil*. Stanford, CA: Stanford University Press.

Mainwaring, Scott and Timothy R. Scully (1995). "Building Democratic Institutions: Party Systems in Latin America". In: Stanford University Press. Chap. Introduction: Party systems in Latin America, pp. 1–34.

Mair, Peter (1997). *Party System Change: Approaches and Interpretations*. Oxford University Press.

Markowski, Radoslaw (2008). "The 2007 Polish Parliamentary Election: Some Structuring, Still a Lot of Chaos". In: *West European Politics* 31.5, pp. 1055–1068.

McMenamin, Iain and Anna Gwiazda (2011). "Three Roads to Institutionalisation: Vote-, Office- and Policy-seeking Explanations of Party Switching in Poland". In: *European Journal of Political Research* 50.6, pp. 838–866.

Mershon, Carol and Olga Shvetsova (2008). "Parliamentary Cycles and Party Switching in Legislatures". In: *Comparative Political Studies* 41.1, pp. 99–127.

Offe, Claus (1993). "The Politics of Social Policy in East European Transitions: Antecedents, Agents, and Agenda of Reform". In: *Social Research* 60.4, pp. 649–684.

Padgett, John F. and Christopher K. Ansell (1993). "Robust Action and the Rise of the Medici, 1400–1434". In: *American Journal of Sociology* 98.6, pp. 1259–1319.

Poole, Keith T. and Howard Rosenthal (1997). *Congress: A Political Economic History of Roll Call Voting*. Oxford, UK: Oxford University Press.

Powell, Eleanor Neff and Joshua A. Tucker (2014). "Revisiting Electoral Volatility in Post-Communist Countries: New Data, New Results and New Approaches". In: *British Journal of Political Science* 44.1, pp. 123–147.

Robins, Garry, Philippa Pattison, and Jodie Woolcock (2004). "Missing Data in Networks: Exponential Random Graph (p^*) Models for Networks with Non-respondents". In: *Social Networks* 26.3, pp. 257–283.

Sarkar, Purnamrita and Andrew W. Moore (2005). "Dynamic Social Network Analysis using Latent Space Models". In: *SIGKDD Explorations* 7.2, pp. 31–40.

Schattschneider, E. E. (1942). *Party Government*. New York, NY: Farrar and Rinehart.

Schweinberger, Michael and Tom A. B. Snijders (2003). "Settings in Social Networks: A Measurement Model". In: *Sociological Methodology* 33.1, pp. 307–341.

Sewell, Daniel K. and Yuguo Chen (2015). "Latent Space Models for Dynamic Networks". In: *Journal of the American Statistical Association*.

Shabad, Goldie and Kazimierz M. Słomczynski (2004). "Inter-Party Mobility among Parliamentary Candidates in Post-Communist East Central Europe". In: *Party Politics* 10.2, pp. 151–176.

Shortreed, Susan, Mark S. Handcock, and Peter D. Hoff (2006). "Positional Estimation Within a Latent Space Model for Networks". In: *Methodology* 2.1, pp. 24–33.

Słomczynski, Kazimierz M. and Goldie Shabad (1996). "Systemic Transformation and the Salience of Class Structure in East Central Europe". In: *East European Politics & Societies* 11.1, pp. 155–189.

Snijders, Tom A. B., Gerhard G. van de Bunt, and Christian Steglich (2010). "Introduction to Stochastic Actor-Based Models for Network Dynamics". In: *Social Networks* 32.1, pp. 44–60.

Stan Development Team (2013). *Stan: A C++ Library for Probability and Sampling*. Version 2.2.0.

Szczerbiak, Aleks (2013). "Poland (Mainly) Chooses Stability and Continuity: The October 2011 Polish Parliamentary Election". In: *Perspectives of European Politics and Society* 14.4, pp. 480–504.

Tavits, Margit (2005). "The Development of Stable Party Support: Electoral Dynamics in Post-Communist Europe". In: *American Journal of Political Science* 49.2, pp. 283–298.

Tavits, Margit and Taavi Annus (2006). "Learning to Make Votes Count: The Role of Democratic Experience". In: *Electoral Studies* 25.1, pp. 72–90.

Tucker, Joshua A. (2006). *Regional Economic Voting: Russia, Poland, Hungary, Slovakia and Russia, 1990-99*. Cambridge, UK: Cambridge University Press.

van Biezen, Ingrid, Peter Mair, and Thomas Poguntke (2012). "Going, going, ... gone? The Decline of Party Membership in Contemporary Europe". In: *European Journal of Political Research* 51.1, pp. 24–56.

Ward, Michael D., John S. Ahlquist, and Arturas Rozenas (2013). "Gravity's Rainbow: A Dynamic Latent Space Network for the World Trade Network". In: *Network Science* 1.1, pp. 95–118.

Wasserman, Stanley and Katherine Faust (1994). *Social Network Analysis: Methods and Applications*. Cambridge, UK: Cambridge University Press.

Whiteley, Paul F. (2011). "Is the Party Over? The Decline of Party Activism and Membership Across the Democratic World". In: *Party Politics* 17.1, pp. 21–44.

Zielinski, Jakub, Kazimierz M. Słomczynski, and Goldie Shabad (2005). "Electoral Control in New Democracies: The Perverse Incentives of Fluid Party Systems". In: *World Politics* 57.3, pp. 365–395.